Course Descriptions

Spring 2019

Department of Mathematics
University of Illinois

MATH 501: ABSTRACT ALGEBRA II

WILLIAM J HABOUSH

I will follow the posted syllabus rather faithfully. I will begin with a discussion of modules over noncommutative rings. Then I will discuss categories and I will prove Yoneda's lemma. I like to explain how Yoneda's lemma is a general proof that definitions made by means of universal mapping properties work. I will then explain how many of the constructions to which the course is devoted are applications of Yoneda's lemma.I also like to discuss adjoint functors and to apply it in discussing such notions as free objects. I will certainly discuss projective and injective modules and exact sequences and related matters.I like to cover Wedderburn theory with some applications to representation theory and to devote a great deal of attention to multilinear algebra and to exterior and summetric algebras. I will try to cover some homological algebra and algebraic geometry. I will use Advanced Modern Algebra by Joseph Rotman as a reference and for its exercises.

E-mail address: haboush@math.uiuc.edu

Math 511
Professor Dodd

This comps course provides an introduction to classical algebraic geometry, using Shavarevich's textbook as a guide. The material will prepare interested students to go on to work towards more advanced topics in algebraic geometry, namely the further courses in scheme theory. We'll discuss the basic notions concerning quasi-projective varieties over an algebraically closed field, such a irreducibility and smoothness, and prove basic results such as Bezout's theorem, reviewing the relevant commutative algebra along the way.

Math 519: Differentiable Manifolds II
Professor Hirani

Riemannian manifolds, affine connections, vector bundles, connections on a vector bundle, curvature, geodesics, parallel transport, Gauss-Bonnet Theorem, vector-valued forms, characteristic classes, Pontrjagin classes, Euler class, Chern classes. Other topics may include principal bundles and connections on principal bundles. The primary references will be Riemannian Geometry by Gallot, Hulin, Lafontaine and Differential Geometry by Tu.

ALGEBRAIC TOPOLOGY I

Igor Mineyev. Math 525, Spring 2019.
`https://faculty.math.illinois.edu/~mineyev/class/19s/525/`
Textbook: *Algebraic topology*, by Allen Hatcher. Freely available online at
`www.math.cornell.edu/~hatcher/AT/ATpage.html`

The tentative syllabus.
Fundamental group and covering spaces.

(1) Definition of the fundamental group.
(2) Covering spaces and lifts of maps.
(3) Computing the fundamental group via covering spaces.
(4) Applications, such as the Fundamental Theorem of Algebra and the Brouwer fixed point theorem in 2d.
(5) Deforming spaces: retraction and homotopy equivalence.
(6) Quotient topology and cell complexes.
(7) Homotopy extension property and applications to homotopy equivalence.
(8) Fundamental groups of CW complexes.
(9) Van Kampen's Theorem.
(10) Covering spaces and subgroups of the fundamental group.
(11) Universal covers.
(12) The definitive lifting criterion, classification of covering spaces.
(13) Covering transformations and regular covers.

Homology.

(14) Delta complexes and their cellular homology.
(15) Singular homology.
(16) Homotopic maps and homology.
(17) The long exact sequence of the pair.
(18) Relative homology and excision.
(19) Equality of cellular and singular homology.
(20) Applications, such as degree of maps of spheres, invariance of dimension, and the Brouwer fixed point theorem.
(21) Homology of CW complexes.
(22) Homology and the fundamental group: the Hurewicz theorem.
(23) Euler characteristic.
(24) Homology with coefficients.
(25) Intro to categories and axiomatic characterization of homology theories.
(26) Further applications, such as the Jordan curve theorem, wild spheres, invariance of domain.

MATH 527: HOMOTOPY THEORY

Instructor: Dominic Culver

Prerequisites: Math 525 and 526 or their equivalent.

Texts: No required textbooks, I will provide a list of useful references later.

Course description: This course will be an introduction to stable homotopy theory. A major problem in algebraic topology is to compute the homotopy groups of spheres, $\pi_*(S^n)$. This problem is rather intractable, and one simplification that can be made is to compute the *stable homotopy groups*: these are obtained by regarding two maps $S^n \to S^{n+k}$ as equivalent if they are homotopic after some number of suspensions. The goal of this course is develop the necessary techniques to study stable homotopy groups. We will accomplish this by taking a careful look at the following three problems, which were historically important to the development of the subject:

(1) Hopf invariant 1 problem: how many maps $S^{2n-1} \to S^n$ of Hopf invariant 1 are there?
(2) Vector fields on spheres: Given a sphere S^n, how many linearly independent vector fields can we put on it?
(3) Image of the J-homomorphism: What is the image of Whitehead's J-homomorphism $J : \pi_*O \to \pi_*^s$.

Another goal of this course will to give students a fluency in spectral sequence computations. The course will roughly go as follows.

(1) Review of the homotopy theory of spaces.
(2) Serre spectral sequence and Serre's finiteness theorem.
(3) Freudenthal suspension theorem
(4) Cohomology theories and the stable homotopy category.
(5) Cohomology operations, Steenrod algebra and its dual.

(6) Adams spectral sequence, computation of stable homotopy groups
(7) topological K-theory and its applications

Time permitting we will also cover

(1) Milnor's and Quillen's theorem on complex cobordism.
(2) Brown-Peterson spectrum and the Adams-Novikov spectral sequence.
(3) Periodicity phenomena in the stable homotopy of the sphere, basic chromatic homotopy theory.

Spring 2019

Math 530 Algebraic Number Theory

Instructor: Duursma, Altgeld Hall 303.

Textbooks (we will use both):

Number Fields, Daniel A. Marcus. (Springer 2018 second edition, free download at springer from illinois account)

Algebraic Theory of Numbers, Pierre Samuel. (Dover 2008)

Supplemented with notes and selected other material.

Prerequisites: Math 417 and Math 500.

Topics Covered:

1. Algebraic Background – Review norm, trace, discriminant, different, integrality, noetherian; Finitely generated torsion-free modules over a PID.
2. Basics – Number fields, rings of integers being Dedekind domains, integral bases, quadratic and cyclotomic fields.
3. Global theory – Lattices in $\mathbb{R}^n$, unit theorems, finiteness of class numbers, examples of computing class numbers using Minkowski bound.
4. Local theory – Completions of $\mathbb{Q}$ (and number fields), Hensel's Lemma with application to nonsolvability of Diophantine equations.
5. Decomposition of Primes – Kummer's Lemma, inverse different, norm of ideals, discriminant, decomposition group, inertia group, Frobenius automorphism, application to quadratic reciprocity.
6. Analytic Methods – Zeta functions of number fields, Dirichlet L-functions, $L(1, \chi)$ for a quadratic character χ.

SPRING 2019, MATH 532, ANALYTIC THEORY OF NUMBERS II : MULTIPLICATIVE NUMBER THEORY

INSTRUCTOR: ALEXANDRU ZAHARESCU

Math 532, TR 12:30 - 1:50 PM

In this course we will discuss ideas from multiplicative number theory centered on $L-$functions and their applications. We first cover some key chapters from Davenport's book. We will assume a certain level of familiarity with the Riemann zeta function and Dirichlet $L-$functions so that these chapters can be covered at a sustained pace. Then we will go in several different directions: a presentation of some more general classes of $L-$functions, a presentation of the resonance method and its applications, and a description of technics using mollifiers and some recent developments.

Prerequisite: MATH 531.

Recommended Textbook:

Harold Davenport, Multiplicative number theory. Third edition. Graduate Texts in Mathematics, 74. Springer-Verlag, New York, 2000. xiv+177 pp. ISBN: 0-387-95097-4

There will be no exams. Students registered for this course will be expected to give a couple of lectures on some topics related to the content of the course. In addition some homework problems will be assigned.

Office hours by appointment.

Office: 449 Altgeld Hall.

E-mail: *zaharesc@illinois.edu*

Math 553
Professor Rapti

Basic introduction to the study of partial differential equations; topics include: the Cauchy problem, canonical forms, the method of characteristics, the wave equation, the heat equation, Laplace's equation, Sturm-Liouville problems and separation of variables, harmonic functions, potential theory, Fourier series, the Dirichlet and Neumann problems, and Green's functions. Prerequisite: Consent of instructor.

Math 540 Real Analysis I, Section B1

Spring 2019, MWF 10:00–10:50am, 243 Mechanical Eng. Bldg.

This is an introductory course in measure theory. For the syllabus see: https://math.illinois.edu/resources/department-resources/syllabus-math-540

Prerequisites: Ma 447 or equivalent. Basic set theory, see chapter 0 of Folland's Analysis.

Textbook: G. B. Folland, Real Analysis, John Wiley & Sons. Additional references are Royden's Real Analysis and Rudin's Real and Complex Analysis.

Contact info:

Instructor: M. Burak Erdoğan
E-mail: berdogan@illinois.edu
Office: 347 Illini Hall
Webpage: https://faculty.math.illinois.edu/∼berdogan/

Math 541-Functional Analysis

Instructor/time: Marius Junge, MWF, 12:00 am to 12:50 am.

Description: After a review of basic principles in measure theory, we will discuss the five highlights in functional analysis related to the Hahn-Banach theorem and Baire's category theorem. The rest of the course is devoted to show how these principles and basic machinery from functional analysis can be used in different applications. In fact, we will provide peek views to operator theory, distribution theory and PDE, and quantum information.

Book: John Conway: A Course in Functional Analysis, Graduate Texts in Mathematics, 96. Springer-Verlag, New York, 1990. ISBN: 0-387-97245-5

Math 561 - Spring 2019
Professor Dey

Course Description: This is the first half of the basic graduate course in probability theory. The goal of this course is to understand the basic tools and language of modern probability theory. We will start with the basic concepts of probability theory: random variables, distributions, expectations, variances, independence and convergence of random variables. Then we will cover the following topics: (1) the basic limit theorems (the law of large numbers, the central limit theorem and the large deviation principle); (2) martingales and their applications. If time allows, we will give a brief introduction to Brownian motion ans Stein's method for normal approximation.

Textbook: Richard Durrett: Probability: Theory and Examples (4th edition), Cambridge University Press, 2010 (ISBN-13: 978-0521765398, ISBN-10: 0521765390). Version 4.1 of the book is freely available from the Author's website and course notes will be provided.

Prerequisite: The prerequisite for Math 561 is Math 540 - Real Analysis I. We will review measure theory topics as needed. Math 541 is nice to have, but not necessary.

Grading Policy: Tentatively 40% of your grade will depend on homework assignment, 30% will depend on the in class midterm test and 30% on the take home final exam.

Math 571

Professor Walsberg

In the 1930's Godel proved two fundamental theorems of logic: the completeness and incompleteness theorems. Model theory studies the implications of Godel's theorems for particular situations in mathematics. In this course we will discuss Model theory at an introductory graduate level. In particular we will prove Morley's famous categoricity theorem, which marked the beginning of modern model theory. Familiarity with the basic syntax and semantics of first order logic will be assumed. Students who haven't taken any course in logic can easily acquire this by reading the first pages of any text in logic, for example, Lou van den Dries's or Anush Tserunyan's notes available on their respective websites.

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF MATHEMATICS
Course Description — SPRING 2019

MATH 582

STRUCTURE OF GRAPHS

Instructor: A. Kostochka, 234 Illini Hall, 265-8037, kostochk@illinois.edu.

TOPICS: This is a companion course to Math 581 — Extremal Graph Theory. The two courses are independent. *Structure of Graphs* includes topics drawn from the following (not all will be covered).

Elementary Structural Concepts — structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs. Graph packings and equitable colorings.

The reconstruction problem — is G reconstructible from the deck of subgraphs obtained by deleting a single vertex? ... a single edge?

Connectivity — min-max relations for connectivity and branchings, structure of k-connected graphs.

Cycles — Hamiltonian cycles and circumference in graphs and digraphs.

Topological Graph Theory — embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, proof of Kuratowski's Theorem, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces. Using discharging for coloring problems on surfaces.

Joins and flows — the language of conservative weightings for finding maximal joins and minimum T-joins, cycle covers and nowhere-zero flows.

Graph Minors — treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has infinitely many minimal forbidden minors), forbidden and forced minors.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The problems require proofs related to or applying results from class.

PREREQUISITES: Familiarity with elementary graph theory. Either of Math 580 and Math 412 provides sufficient preparation. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, Graph Theory, or the Math 412 text: West, Introduction to Graph Theory (Prentice Hall, 2001, first seven chapters). Important results needed from elementary graph theory will be reviewed.

TEXT: D. B. West, The Art of Combinatorics, Volume II: Structure of Graphs. For some topics, instructor's supplements will be provided.

Math 595

Professor Balog

Many important theorems and conjectures in combinatorics, such as the theorem of Szemer\'edi on arithmetic progressions and the Erdos-Stone Theorem in extremal graph theory, can be phrased as statements about families of independent sets in certain uniform hypergraphs.

These hypergraphs have a clustering phenomena, which can be summarized in a general theorem, called as Container Theorem, and the method is the container method. The method seems to be surprisingly applicable for enumerating problems, extremal questions in random environment, and proving the existence of some combinatorial structures.

The course will discuss the container method, its variant, and many of its application, including in probability theory, additive number theory, discrete geometry, combinatorics and graph theory.

Grade requirement: Students should demonstrate understanding the course material, which could include read some related paper, typing up some class notes, or do some homework assignments.

Prerequisities: 412 and 413, or 580, or equivalent.

Algebraic and Differential Topology in Data Analysis (ADTDA)

Yuliy Baryshnikov

The course will cover some recent applications of topology and differential geometry in data analysis.

Tools of differential and algebraic topology are starting to make noticeable impact in the area of data sciences, where the mathematical apparatus thus far was dominated by the ideas from statistical learning, computational linear algebra and high-dimensional normed space theories.

While the research community around ADT topics in data analysis is lively and fast-growing, the area is somewhat sparsely represented in campus syllabi. I wanted to test ground whether there is a constituency for a course covering some of it (with the choice of topics, of course, tilted towards what I find interesting).

There are several courses on Data Analysis on campus (IE 531, CSE 448, ECE 566); however they have thematically virtually no overlap with the proposed syllabus.

I expect besides Math students interested in applications of geometric and topological tools, some participation from CoE students.

Syllabus [43][1]

1. Tools from algebraic topology [4]
   - 1.1. Homotopy equivalence
   - 1.2. Simplicial homology
   - 1.3. Nerve lemma, Dowker's theorem
   - 1.4. **Applications**:
     Sketches: Merge trees; Reeb graphs; skeletonizations
2. Tools from Hodge theory [4]
   - 2.1. Laplacians, eigenvalues, eigenfunctions
   - 2.2. Interplay between geometry and spectral properties. Cheeger inequality
   - 2.3. Expanders
   - 2.4. **Applications**:
     Synchronization, Clustering
3. Tools from differential topology [7]
   - 3.1. Basic notions: topological spaces, manifolds, simplicial complexes.
   - 3.2. Transversality.
   - 3.3. Sard's theorem.

[1] [*] indicates hours allocated

    3.4. Whitney's embedding theorem.
    3.5. **Applications**:
    Dimensionality reduction: embedding based tools (PCA, Nonlinear PCA; Random Projections; Multidimensional Scaling); embedding generating tools (Isomap; Eigenmap; Diffusion Maps; LLE).
4. Topological Approximations [7]
    4.1. Vietoris-Rips and Čech complexes.
    4.2. Topology reconstruction from dense samples: Hausmann-Latscher
    4.3. Topology reconstruction from random samples: Niyogi-Smale-Weinberger
    4.4. **Applications**:
    Netflix problem complexes
    Cech complexes and their topology in robotic and neurophysiology.
    Merge trees in time series analysis
5. Euler calculus [6]
    5.1. Set algebras and valuations
    5.2. Hadwiger and McMullen's theorems
    5.3. Integration with respect to Euler characteristics
    5.4. Fundamental Kinematic Formula
    5.5. **Applications**:
    Gaussianity tests for random fields
    Topological Sensor Network
6. Topological Inference [6]
    6.1. Persistent Homology
    6.2. Algorithms
    6.3. Stability
    6.4. **Applications**:
    Image patches spaces
    Textures and characterization of materials
7. Aggregation [4]
    7.1. Spaces with averagings
    7.2. Arrow theorem and Topological Social Choice
    7.3. Aggregation in CAT(0) spaces
    7.4. **Applications**:
    Consensus in phylogenetic analysis
8. Clustering [3]
    8.1. Basic clustering tools
    8.2. Kleinberg's Impossibility theorem.
    8.3. Carlsson-Memoli functorial approach to clustering.
9. Students presentations [2]

With exception of background material, covered in the books below, the course will rely mainly on the recent papers.

R Ghrist, Elementary Applied Topology, Createspace, 2014
A Hatcher, Algebraic Topology, CUP, 2002
V Prasolov, Elements of Combinatorial and Differential Topology, AMS, 2006

The students will be given 4-5 homework assignments, to practice for the final.
Each student will be expected to either present a paper (from a sample provided by me), or to run a computational project (also designed by me). The presentations account for 40% of the overall score; the final exam, 60%.

Math 595: Algebraic curves and surfaces

Spring 2019

Time: TBA (second half of term).

Location:TBA.

Instructor: Emily Cliff.

Office hours: TBA.

Prerequisites: It will be assumed that most students will be (at least moderately) familiar with the material in Chapters II and III of Hartshorne's *Algebraic Geometry*, that is, foundations of schemes and their cohomology. However, if you are interested and motivated, and willing to do some independent reading, I will work with you to make it possible for you to follow and benefit from the course. In that case please contact me before registering.

Course content: We will cover selected topics from Chapters IV and V of Hartshorne's *Algebraic Geometry*. The idea is to work with applications of the concepts from Chapters II and III to concrete examples of curves and surfaces.

More specifically, we will begin our study of curves with a discussion of the Riemann-Roch theorem. We will then discuss two ways of explicitly representing and studying curves: as a branched covering of $\mathbb{P}^1$, or as a curve embedded in a higher dimensional projective space. We will also discuss topics in the geometry of algebraic surfaces. In particular we will study ruled surfaces, and see how we can use our knowledge of curves to gain a better understanding of these surfaces. We will also look at the example of non-singular cubic surfaces in $\mathbb{P}^n$.

These topics are not set in stone; students with particular interests building on (or confusions relating to) the theory of schemes should feel free to suggest topics that are exciting or helpful to them, before or during the term.

Grading: Grades will be assigned mostly on the basis of attendance and participation. Problem sets will be assigned; they will not be mandatory but students should not expect to internalize much of the material without engaging with examples such as those on the problem sets.

University of Illinois
Department of Mathematics
257 Altgeld Hall
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C O U R S E   D E S C R I P T I O N

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Spring 2019

MATH 595

S M O O T H   A N D   E T A L E   E X T E N S I O N S

Prof. S. P. Dutta
Room:
TR from 12:30 am to 1:50 am

This is a one-semester course covering several areas in commutative algebra and algebraic geometry on Smooth and Etale extensions. Our main focus will be on the following topics: Weierstrass Preparation Theorem; structure theorem for complete local rings; Zariski's Main Theorem; unramified, étale and smooth extensions and their corresponding structure theorems; Henselian Rings and Henselization; Artin's approximation theorem; Hochster's construction of big Cohen-Macaulay modules and finally Swan's exposition of Popescu's proof of Artin's conjecture on smooth extensions.

The following book covers several topics (not all) mentioned above.

Text: Birger Iversen - Generic local structure in commutative algebra--Lecture notes in Math 310, Springer Verlag, berlin Heidelberg, New York.

SPRING 2019
MATH 595
BRIDGELAND STABILITY CONDITIONS AND APPLICATIONS

Time: TBA
Instructor: Sheldon Katz
Prerequisites: Algebraic Geometry II (Math 595AG2) or equivalent, or permission of the instructor

This first half minicourse introduces the topic of Bridgeland Stability together with applications.

In 2002, Tom Bridgeland proposed his notion of stability as a precise mathematical formulation of the notion of Pi-stability in string theory. The notion proved to be the 'correct' one, and a theory was born. Applications include moduli problems arising in algebraic geometry and the theory of quiver represention; birational geometry; and connections to physics.

The course will begin with "Lectures on Bridgeland Stability," Macri and Schmidt, arXiv:1607.01262[math.AG]. The lecture notes begin with the classical notion of stability of vector bundles. Then Bridgeland stability is introduced and applied to surfaces and threefolds, with some applicationns.

Additional topics will be covered as time allows, possibly including other moduli spaces in algebraic geometry and quiver representation theory; birational geometry; and connections to string theory and mirror symmetry.

Math 595 MNA Spring 2019
Methods in Nonlinear Analysis and Applications to Differential Equations

Class time: TR 2 – 3:20pm
Lecturer: Eduard Kirr, e-mail: ekirr@illinois.edu
Description: The first part of the course will focus on fixed point theorems based on *degree theory* and their applications to differential equations. The (Brouwer) degree for maps between finite dimensional spaces has deep roots in homotopy and homology and you might have encountered it when studying these theories. While I will briefly discuss how it is introduced there, I will present a more axiomatic view of degree theory which can be generalized to (nonlinear) maps between infinite Banach spaces having certain compactness properties (completely continuous, properly bounded, etc.). I will discuss its powerful consequences: *Brouwer, Borsuk-Ulam, Ham and Sandwich Theorems* in finite dimensions and the *Leray-Schauder type fixed point theorems* in infinite dimensions. Their applications to modern *local and global bifurcation theories* and to solutions of differential equations will be emphasized. This part of the course will culminate with the contribution made by the degree theory in understanding the *collapse of Tacoma-Narrows bridge,* a phenomenon that was not predicted by laboratory simulations or the structural and dynamical stability theory preceding its construction.

The second part of the course will focus on *contraction principle* and *variational methods.* While independent, this part will extend some of the results in the first part to non-completely continuous (non-compact) maps. Metric and Banach spaces will be reviewed and *Calculus in Banach spaces* will be introduced before proving the Banach fixed point theorem and its consequences e.g., the implicit function theorem (IFT) in (infinite) dimensional Banach spaces. The application of the contraction principle to existence, uniqueness, continuous dependence of data and stability for solutions of evolution equations (including systems of ordinary and partial differential equations) will be briefly discussed while the applications of the IFT to *Lyapunov-Schmidt decomposition* and *local bifurcation theory* will be presented in detail with examples from nonlinear optics and statistical physics. Extensions of the local results to non-perturbative regimes via the *global bifurcation theory* for real analytical maps will also be discussed. The variational methods will be based on the rigorous calculus in Banach spaces which replaces the more common but ad-hoc "calculus of variation" in which the definition of "variation" seems to change from problem to problem. Moreover, when it comes to applications in finding certain equilibria or periodic solutions in partial different equations, the variational methods have to cope with non-convex functional and non-compact constraints. We will discuss how to compensate for these shortcomings via *Rellich or concentration compactness* and then apply the classical theory which of course will be introduced.

References: I will follow my own notes (posted online) based on the following references:

1. <u>Topics in Nonlinear Functional Analysis</u> by L. Nirenberg.

2. <u>Leray-Schauder degree: a half century of extensions and applications</u> by J. Mawhin in Topol. Methods Nonlinear Anal. Volume 14, Number 2 (1999), 195-228.

3. <u>Partial Differential Equation</u> by Lawrence C. Evans.

4. <u>Long time dynamics and coherent states in nonlinear wave equations</u> by E. Kirr in Recent Progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science, R. Melnik, R. Makarov, J. Belair, Eds. in Fields Institute Communications, 2017, pp. 59–88.

5. <u>Large Torsional Oscillations in a Suspension Bridge: Multiple Periodic Solutions to a Nonlinear Wave Equation</u> by K.S. Moore in SIAM J. Math. Anal. Vol. 33, No. 6, pp. 1411-1429

Grading Policy: There are no homework assignments or exams for this course. The participants will be asked to make a presentation on the applications of these techniques to a nonlinear differential equation, preferably from their own research area. Grades will be based on class activity, and on the quality of the presentation.

GEOMETRIC REPRESENTATION THEORY, ENUMERATIVE GEOMETRY AND QUIVERS.

KEVIN MCGERTY

1

Since their introduction in the seminal paper [N1], Nakajima's quiver varieties have played a central role in geometric representations theory. They provide a rich class of examples of smooth holomorphic symplectic varieties (indeed hyper-Kähler manifolds) and thus are an important testing-ground for the emerging field of "symplectic representation theory". This course will review some recent developments in this direction, including categorification of Nakajima's cohomological constructions of representations of Lie algebras, and enumerative topics such as the calculation of the quantum D-module for quiver varieties following the work of Maulik and Okounkov [MN]. The list below gives an outline of topics to be covered, which is probably over-ambitious, and the focus of the course will be influenced by the interests of the audience.

Topics:

(1) Review of quiver varieties: construction, Poisson geometry etc.
(2) Quantization of quiver varieties via quantum Hamiltonian reduction, and brief discussion of categories of sheaves of modules for such quantizations.
(3) Categorification of Nakajima's actions on cohomology of quiver varieties.
(4) Centre constructions and ring structure on cohomology of moduli space of framed instantons over the affine plane.
(5) Review of quantum cohomology
(6) R-matrices and Yangians
(7) Maulik-Okounkov: stable envelopes and calculation of quantum cohomology of quiver varieties via Yangians.
(8) Cohomological Hall algebras and relation to the Maulik-Okounkov Yangian.

References

[N1] H. Nakajima; *Quiver varieties and Kac-Moody algebras.* Duke Math. J. 91 (1998), no. 3, 515–560.

[MN] D. Maulik,; A. Okounkov; *Quantum Groups and Quantum Cohomology*, arXiv:1210:1287.

[SVV] Shan, P.; Varagnolo, M.; Vasserot, E. On the center of quiver Hecke algebras. Duke Math. J. 166 (2017), no. 6, 1005–1101.

MATHEMATICAL INSTITUTE, OXFORD AND UNIVERSITY OF ILLINOIS.

Date: October 16, 2018.

MATH 595: COHOMOLOGY OF SCHEMES, SPRING 2019

Course Meets: First half of the semester, TuTh 11am-12:20pm

Instructor: Thomas Nevins (nevins@illinois.edu)

Prerequisites: Math 500; Math 510 or Math 511 or some basic knowledge of algebraic varieties.

Course Web Page: http://www.math.uiuc.edu/∼nevins/courses/spr19/m595.html

The course will present an introduction to the cohomology of vector bundles and more general (coherent or quasicoherent) sheaves on algebraic varieties and more general schemes.

Many geometric and topological problems about algebraic varieties (as well as real and complex manifolds and other kinds of topological spaces) can be interpreted in terms of vector bundles or sheaves on those varieties; solutions to the problems then typically involve computing things about the sections of those vector bundles/sheaves, and such computations typically rely on information about cohomology. Thus, vector bundle/sheaf cohomology is an indispensible tool in algebraic geometry and parts of complex geometry, algebraic number theory, homotopy theory, and more.

This is a crash course in sheaf cohomology with an emphasis on Čech cohomology as both a definition and a computational tool. We will develop the theory but also carry out a lot of calculations. The main goal of the course: students should finish the course able to use cohomology to solve geometric problems.

In light of the course goal, the course meetings will involve a mix of lecture and problem-solving sessions.

Mathematics 595 — Higher category theory and quasicategories

(2:00 MWF in 243 Altgeld, first half of semester)

Instructor: Charles Rezk

Course description:
Higher category theory is the study of structures which are like categories, but are "higher-dimensional": while a category has objects (0 dimensions), and morphisms between objects (1 dimensions), higher dimensional analogues are allowed to have morphisms between morphisms (2 dimensions), and so on.

The goal of this course is to descibe an approach to this called *quasicategories*. These were invented by Boardman and Vogt, and were developed further by André Joyal (in various papers and unpublished preprints) and Jacob Lurie (in his book *Higher topos theory*, where he calls them ∞-categories).

The goal of this course is to give a brief and *accessible* introduction to this theory. That is, we imagine that we are familiar with classical category theory, and we are confronted with the strange new notion of a quasicategory. We will attempt to develop basic concepts and results for quasicategories by analogy with what we know about classical categories.

This is, of course, not necessarily straightforward. In order to proceed, we will need to develop "simplicial homotopy theory" and the theory of "model categories". These are not prerequisites: they will be introduced and developed through the course.

Prerequisites: Some familiarity with the basic notions of classical category theory is necessary (e.g., functors, natural transformations, limits and colimits, etc.)

Familiarity with basic algebraic topology (e.g., fundamental group and singular homology, as in Math 525), or homological algebra, will be helpful, but not essential.

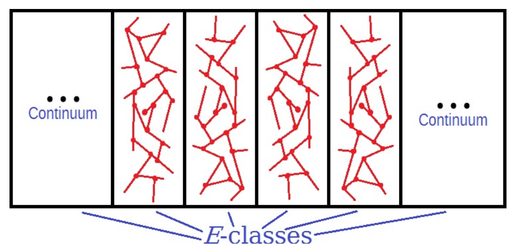
Texts: The main text are notes that I began writing the previous time I taught this: they are available from my homepage.

COUNTABLE BOREL EQUIVALENCE RELATIONS

MATH 595 — Anush Tserunyan — 2019 SPRING

Location TBA — TueThu 11:00–12:20

An equivalence relation E on a Polish space X (e.g., $X := \mathbb{R}, L^1(\mathbb{R}), \mathbb{N}^{\mathbb{N}}$) is *Borel* if it is a Borel subset of X^2, and it is *countable* if each E-class is countable. Countable Borel equivalence relations (CBERs) naturally arise as orbit equivalence relations of Borel actions of countable groups (e.g., $\mathbb{Q} \curvearrowright \mathbb{R}$ by translation). From another, rather combinatorial, angle, a CBER E can always be viewed as the connectedness relation of a locally countable Borel graph (e.g., take E as the set of edges). These connections between equivalence relations, group actions, and graphs create an extremely fruitful interplay between descriptive set theory, ergodic theory (probability measure preserving actions), measured group theory, probability, descriptive graph combinatorics (Borel colorings and matchings), and geometric group theory (trees and quasi-isometry).



This course will feature this interplay. On one hand, we will learn some tools from each of the aforementioned subjects to analyze the structure of CBERs. On the other hand, we will utilize the basic theory of CBERs to prove some well-known results in those subjects.

Prerequisites: Being comfortable with

- basic *soft analysis* (basic pointset topology and metric spaces),
- basic *measure theory* (Borel and measurable sets and functions),
- *group actions* (definition, orbits),
- and *graph-theoretic terminology* (cycles, connectedness, trees).

No knowledge of descriptive set theory is required, we will cover what we need on the fly.

Coursework: Occasional homework, as well as one in-class presentations of a relevant topic/paper.

References: We will use parts of

- "The Theory of Countable Borel Equivalence Relations" by A. Kechris [pdf [1]],
- "Topics in Orbit Equivalence" by A. Kechris and B. Miller [Springer link [2]],
- and "Introduction to Descriptive Set Theory" by A. Tserunyan [pdf [3]].

[1] Online at http://www.math.caltech.edu/~kechris/papers/lectures%20on%20CBER01.pdf

[2] Free for UIUC to download at https://link.springer.com/book/10.1007/b99421

[3] Online at https://faculty.math.illinois.edu/~anush/Teaching_notes/dst_lectures.pdf