Course Descriptions

Fall 2018

Department of Mathematics
University of Illinois

MATH 500

PROFESSOR DUTTA

Basic facts about groups, rings, vector spaces, such as those covered in Math 416 and Math 417 courses, are assumed. Instructors should not spend time on elementary material: the syllabus is quite full. Books that could be used include "Abstract Algebra" by Dummitt and Foote, "Algebra" by Hungerford, and "Advanced Modern Algebra" by Rotman.

1. Group Theory. [Approximately 4.5 weeks]
(a) Isomorphism theorems for groups.
(b) Group actions on sets; orbits, stabilizers. Application to conjugacy classes, centralizers, normalizers.
(c) The class equation with application to finite p-groups and the simplicity of A_5.
(d) Composition series in a group. Refinement Theorem and Jordan-Hölder Theorem. Solvable and nilpotent groups.
(e) Sylow Theorems and applications.

2. Commutative rings and Modules.[Approximately 5 weeks]
(a) Review of subrings, ideals and quotient rings. Integral domains and fields. Polynomial rings over a commutative ring.
(b) Euclidean rings, PID's, UFD's.
(c) Brief introduction to modules (over commutative rings), submodules, quotient modules.
(d) Free modules, invariance of rank. Torsion modules, torsion free modules. Primary decomposition theorem for torsion modules over PID's.
(e) Structure theorem for nitely generated modules over a PID. Application to finitely generated Abelian groups and to canonical form of matrices.
(f) Zorn's lemma and Axiom of Choice (no proofs). Application to maximal ideals, bases of vector spaces.

3. Field Theory. [Approximately 5 weeks]
(a) Prime fields, characteristic of a field.
(b) Algebraic and transcendental extensions, degree of an extension. Irreducible polynomial of an algebraic element.
(c) Normal extensions and splitting fields. Galois group of an extension.
(d) Algebraic closure, existence and uniqueness via Zorn's Lemma. Finite fields.
(e) Fundamental theorem of Galois theory.
(f) Examples of Galois extensions. Cyclotomic extensions.
(g) If time permits, application of Galois theory to solution of polynomial equations, symmetric functions and ruler and compass constructions.

MATH 502

Professor Dodd

This course is an introduction to modern commutative algebra, with the goal of preparing students to take advanced courses in algebraic geometry, number theory, and algebra. We will cover the basics of the subject (as presented in Atiyah-Macdonald) and then go on to discuss homological methods and applications.

Math 506: Representation Theory, Fall 2018

Course meets: Tuesdays & Thursdays 9:30am - 10:50am

Instructor: Chelsea Walton (notlaw@illinois.edu)

Prerequisites: Math 500

Course Web Page: available in August 2018

Homework: There will be 7-8 assignments throughout the semester (~biweekly).

We will use primarily a new textbook "A Tour of Representation Theory" by Martin Lorenz— the preliminary version is available here:

https://www.math.temple.edu/~lorenz/ToR.pdf

As Lorenz remarks:

> "The objective of representation theory is to investigate the different ways in which a given algebraic object—such as an algebra, a group, or a Lie algebra—can act on a vector space. The benefits of such an action are at least twofold: the structure of the acting object gives rise to symmetries of the vector space on which it acts; and, in the other direction, the highly developed machinery of linear algebra can be brought to bear on the acting object itself to help uncover some of its hidden properties. Besides being a subject of great intrinsic beauty, representation enjoys the additional benefit of having applications in myriad contexts other than algebra, ranging from number theory, geometry and combinatorics to general physics, quantum field theory, and the study of molecules in chemistry."

I will cover the representation theory of algebras and of groups in detail (Parts I and II of Lorenz's text), and I will also provide some material on the representation of Lie algebras (Part III) and of Hopf algebras (Part IV) as time permits.

Texts and papers for additional reading will be suggested throughout the semester.

Most of all, I aim to make the course fun and exciting!

MATH 512 Modern Algebraic Geometry
Fall 2018

Instructor: Jeremiah Heller (jbheller@illinois.edu)

Course description: This is an introduction to schemes, roughly along the lines of Hartshorne Chapter 2. Topics covered include:

Sheaves.
Spec of a ring, Zariski topology, Proj of a graded ring.
Basic properties of schemes and morphisms.
Quasi-coherent modules.
Weil & Cartier divisors, Picard group.
Kahler differentials.

Assessment: There will be regular homework and a final project.

FALL 2018
MATH 514
COMPLEX ALGEBRAIC GEOMETRY

Time: 9:30–10:50am TR
Instructor: Sheldon Katz
Text: Hodge Theory and Complex Algebraic Geometry, I, C. Voisin
Prerequisites: MATH 448 or permission of the instructor

This course provides an introduction to complex manifolds, especially compact Kähler manifolds. The class of Kähler manifolds include all smooth projective varieties over the complex numbers but is more general, including for example complex tori of any dimension. The course will culminate with Hodge theory including the Hodge decomposition, which concisely expresses the beautiful interplay between the analytic properties and the topology of Kähler manifolds. Applications will be given.

The material will largely follow the first six chapters of the text

1. Holomorphic functions of many variables
2. Complex manifolds
3. Kähler metrics
4. Sheaves and cohomology
5. Harmonic forms and cohomology
6. The case of Kähler manifolds

The lectures will often deviate from the text and will include material from other sources.

Differentiable Manifolds I (MATH 518)

Fall 2018

Instructor: Anil Hirani, hirani@illinois.edu

Course Outline:

1. **Manifolds:** Definitions and examples including projective spaces and Lie groups; smooth functions and mappings; submanifolds; the inverse function theorem and its applications including transversality; (co)tangent vectors and bundles; Whitney's embedding theorem; manifolds with boundary; orientations.
2. **Exterior Calculus on Manifolds:** Vector fields, flows, and the Lie derivative/bracket; differential forms and the exterior algebra of forms; orientations again; exterior derivatives, Hodge star, contraction, and the Lie derivative of forms; integration and Stokes Theorem.
3. **Applications:** A subset of the following, as time permits: Hodge decomposition; Sard's Theorem; Distributions and the Frobenius Theorem; Lefschetz fixed point theorem; Poincare-Hopf index theorem; DeRham cohomology.

Textbook: Two standard textbooks have been shortlisted and are being evaluated for use in the course.

Grading: Homeworks (25%), Midterm exam (30%), Comprehensive final exam (45%).

Math 526, Topology II (Fall 2018)

Instructor: Pierre Albin

Office: Illini Hall 237

Email: palbin [at] illinois .edu

Lectures: TBA

Office Hours: TBA

Web page: https://faculty.math.illinois.edu/~palbin/Math526.Fall2018/home.html

Text: Hatcher, *Algebraic Topology* available on the author's webpage

Supplementary Texts:
Bredon, *Topology and Geometry*
May, *A concise course in Algebraic Topology* , available on the author's webpage

Assignments: There will be homework each week.You are allowed (and encouraged) to work with other students while trying to understand the homework problems. However, the homework that you hand in should be your work alone. Late homework will not be accepted, but the lowest score will be dropped.

Holidays: Classes begin on August 27 and end on December 12. There will be no classes on:
Labor Day, September 3
Thanksgiving break, November 19 - November 23

Grading percentages:
Problem sets (100%)

Description:
Poincaré's initial analysis situs paper had as its main objective proving an important duality between the dimensions of homology groups of complementary degree. The study of duality in homology naturally led to the definition and study of cohomology groups. These groups have additional structure over the homology groups, namely there is a graded multiplication of cohomology classes that turns the cohomology groups into a ring. We will study this cup product and establish Poincaré duality.

The other main topic of this course is higher homotopy theory. Whereas Math 525 studied the homotopy group of pointed maps from the circle into a topological space (the fundamental group), in this course we will study homotopy classes of maps from higher dimensional spheres. These groups are always Abelian, but are almost always impossible to compute. The techniques invented to study them are at the center of contemporary algebraic topology.

FALL 2018, Math 531: Analytic Theory of Numbers I

Instructor: Alexandru Zaharescu
Office: 449 Altgeld Hall
Phone: 265-5439
E-mail: zaharesc@illinois.edu

Lectures: TR 12:30-1:50pm

Course Description. We will follow closely Professor Hildebrand's lecture notes. Topics will include:

(1) Arithmetic functions.
(2) Elementary theorems on the distribution of primes.
(3) Dirichlet series and Euler products.
(4) Properties of the Riemnann zeta function.
(5) Analytic proof of the Prime Number Theorem.
(6) Dirichlet's theorem on primes in arithmetic progressions.

Prerequisite: MATH 448 and either MATH 417 or MATH 453.

Grading Policy: Comprehensive final exam: 35%; Two midterm exams: 2×25=50%; Homework: 15%.

Recommended Textbooks:

Main reference: A. J. Hildebrand, Introduction to Analytic Number Theory, available on Professor Hildebrand's webpage.
Additional reference: T. M. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1st ed. 1976.

GENERAL TOPOLOGY

Igor Mineyev. Math 535, Fall 2018, MWF 2pm.
`www.math.uiuc.edu/~mineyev/class/18f/535/`

This course will present foundations of general topology, which is a subject used in many areas of mathematics. We will roughly follow Part 1 of the textbook and discuss additional topics from algebraic topology and functional analysis. Any other information will appear on the class website.

Textbook:

- James Munkres, *Topology*, second edition.
- For additional material, feel free to look up the *Algebraic topology* book by Hatcher freely available online.

Consider also taking the topics course that I will be teaching in the Fall 2018: Math 595 *Open problems in group theory and topology* (October 22 - December 12). That course will only run if sufficiently many students sign up.

Math 540-Real Analysis

Instructor: Marius Junge

Math 540 is one of the core courses. As such this course will provide an introduction into Lebesgue theory of measure and integration which is the standard in mathematics for almost 100 years. Some aspects of real analysis are used in different areas of mathematics, ranging from probability theory, combinatorics, functional analysis, number theory to geometry. Another aim of the course is to give a glimpse of interesting topics in analysis which are areas of active research. We follow Roydon's book. The main text provides a stream-lined sketch of the main ideas. The devil is in the details and is in hidden in the exercises.

Book: Real Analysis, Roydon, 4th edition, person modern classics for advanced mathematics series.

Math 542 Complex Variables I Fall 2018

Le chemin le plus court entre deux vérités du domaine réel passe bien souvent par le domaine complexe.

The shortest path between two truths of the real domain quite often passes through the complex domain.

Jacques Salomon Hadamard (1865–1963)

This is the first graduate course in complex analysis. It focuses on the analytic theory of functions of a single complex variable. A highlight is the *Riemann mapping theorem*, which characterizes the domains in the complex plane which can be mapped by a one-to-one analytic function onto the unit disc. We will reach the Riemann mapping theorem about halfway through the course. Other topics to be discussed include: the Cauchy–Riemann equations, Cauchy's integral theory (local and global), Liouville's theorem (bounded analytic functions defined on the entire complex plane are constant), the maximum modulus theorem (the modulus of an analytic function has no interior maxima), meromorphic functions and the classification of singularities, infinite products and the Weierstrass factorization theorem, hyperbolic geometry, Mittag-Leffler's theorem (on the existence of meromorphic functions with prescribed singularities), harmonic functions and the Dirichlet problem, and theorems of Bloch and Picard on the range of analytic functions.

Instructor: Jeremy Tyson, `tyson@illinois.edu`, 244-4132

Homework, exams: There will be weekly homework assignments, two midterm exams, and one final exam.

Course Text: *Complex Analysis: In the Spirit of Lipman Bers*, second ed., by R. E. Rodríguez, I. Kra and J. P. Gilman, Graduate Texts in Mathematics, vol. 245, Springer, 2013

This book is freely available as a Springer E-book through the University Library. For more information and the access link, see

`https://vufind.carli.illinois.edu/vf-uiu/Record/uiu_7039571`

Other recommended texts:

- *An Introduction to Complex Function Theory*, first ed., by B. Palka, Springer, 1991.
- *Complex Analysis*, third ed., by L. V. Ahlfors, McGraw-Hill, 1979.
- *Functions of One Complex Variable I*, second ed., by J. B. Conway, Springer, 1978.
- *Complex Analysis*, by E. M. Stein and R. Shakarchi, Princeton Lectures in Analysis II, Princeton University Press, 2003.

Syllabus

I. Complex numbers, topology of the complex plane, stereographic projection
II. Cauchy–Riemann equations, analytic functions, basic conformal mappings
III. Power series, exponential and logarithm, zeros and poles
IV. Contour integration, local Cauchy theory, Cauchy integral theorem, winding number, homotopy and simple connectivity
V. Liouville's theorem, maximum modulus theorem, Schwarz Lemma
VI. Laurent series, isolated singularities, residues, argument principle, Rouché's theorem
VII. Arzelà–Ascoli theorem, normal families, Riemann mapping theorem
VIII. Conformal maps, hyperbolic geometry, Poincaré metric, Blaschke products
IX. Harmonic functions, Laplace's equation, Poisson integral formula, subharmonic functions, the Dirichlet problem
X. infinite products, Weierstrass factorization theorem, Mittag-Leffler theorem
XI. Bloch's theorem, Picard's Little Theorem, Picard's Great Theorem
XII. (optional) Riemann zeta function, elliptic functions, Weierstrass $\wp$ function

Math 545 Harmonic Analysis

Fall 2018, MWF 9 - 9:50

This is an introductory course in Harmonic Analysis. The emphasis will be on the real variable methods. Some topics we hope to cover include:
- L^p spaces, interpolation, maximal functions.
- The Fourier transform on the circle and on $\mathbb{R}^n$: basic properties, convergence and summability issues.
- Singular integral operators, Fourier multipliers, Mikhlin multiplier theorem, Littlewood-Paley theory.
- Sobolev and Lipschitz spaces, fractional integration, Sobolev embedding.
- Oscillatory integral estimates, Fourier restriction, Bochner-Riesz summability.
- Applications in geometric measure theory such as Kakeya and distance set problems

Prerequisites: A solid knowledge of multivariable calculus. Familiarity with Lebesgue integration, L^p spaces, complex function theory.

Contact info:

Instructor: M. Burak Erdoğan
E-mail: berdogan@math.uiuc.edu
Office: 347 Illini Hall
Webpage: http://www.illinois.edu/∼berdogan (there will be a link to the course webpage)

Textbook: There is no required textbook for this class.
References:
Y. Katznelson, *An introduction to Harmonic Analysis*
J. Duoandikoetxea, *Fourier Analysis*
T. Wolff's Lectures on Harmonic Analysis
(available at www.math.ubc.ca/∼ilaba/wolff/)
E. Stein, *Harmonic Analysis*
C. Muscalu and W. Schlag, *Classical and multilinear harmonic analysis.* Vol 1.

Math 546 Hilbert Spaces
Section xx, Fall 2018

Instructor: Zhong-Jin Ruan
Classroom: xxx Altgeld Hall MWF xxxx
Office: 353 Altgeld Hall
Email: ruan@math.uiuc.edu

This is an introductory course in Operator Theory and Operator Algebras. In this course, we plan to cover

1) Geometric properties of Hilbert spaces
2) Compact and bounded linear operators on Hilbert spaces
3) Basic theory of Banach algebras and C*-algebras
4) Spectral theory of bounded normal operators
5) Unboubded operators on Hilbert spaces.

Prerequisite: Math 541.

Text: A course in functional analysis by John B. Conway,
2ed Edition, Springer

References:

1) A Course in Operator Theory, J. Conway.
2) Fundementals of the Theory of Operator Algebras I.
Kadison and Ringrose
3) Theory of Operator Algebras I. M. Takesaki.

MATH 550

Dynamical Systems I

An introduction to the study of dynamical systems. Considers continuous and discrete dynamical systems: linear and nonlinear differential equations, flows and maps on Euclidean space and other manifolds. Among other things, we will study the existence and uniqueness of solutions, dependence on initial conditions and parameters, linearization, stable and center manifold theorems.
Discrete dynamics includes Bernoulli shifts, elementary Anosov diffeomorphisms and surfaces of sections of flows.
Bifurcation phenomena in both continuous and discrete dynamics will be studied.

Prerequisite: MATH 489 or consent of instructor.

Textbook information is below.

Title: Ordinary Differential Equations with Applications
Author: Carmen Charles Chicone
Series: Texts in Applied Mathematics
Publisher: Springer

Math 554: Linear Analysis and Partial Differential Equations
Fall 2018

Lecturer: Eduard Kirr, e-mail: ekirr@illinois.edu

Room and Time: TBA.

References:
1. Math 554 Notes by E. Kirr, posted online;
2. Partial Differential Equation by Lawrence C. Evans (recommended);
3. Equations of Mathematical Physics by V.S. Vladimirov (recommended);
4. Sobolev Spaces by R. A. Adams (recommended);
5. Semigroups of linear operators by A. Pazy (recommended).

Syllabus: We will basically cover Chapters 5-7 from the reference [2] but use mostly my notes. We will start by relating certain partial differential equations with physical phenomena. Next, we will introduce Sobolev spaces and review certain properties of Banach, Hilbert spaces, as well as of linear operators defined on them. Then, we will use this framework to study both weak and classical solutions of the following second order equations on bounded domains: elliptic equations (Lax-Milgram Theorem, regularity, maximum principles, spectral properties i.e., eigenvalues and eigenfunction); parabolic equations (energy estimates, regularity, maximum principle); hyperbolic equations (energy estimates, propagation of singularities).

We will also develop the semigroup theory and show how it applies to linear evolution equations (such as the parabolic and hyperbolic ones). The theories and the applications encountered in this course will create a strong foundation for studying nonlinear equations and nonlinear science in general.

Prerequisites: Real Analysis (Math 447 or equivalent) and Ordinary Differential Equations (Math 489 or equivalent) or consent of the instructor. Measure Theory (Math 540), Functional Analysis (Math 541) and familiarity with partial differential equations (Math 553) are helpful but not required.

Homework: Assignments will be given approximately every two weeks. Solutions should be written up independently and turned in by the deadline for each assignment.

Grading Policy: Grades will be based on homework and class activity.

MATH 558 Methods of Applied Mathematics Fall 2018

It is impossible to comprehensively cover Applied Mathematics in one semester, or perhaps even in one lifetime. What we will do here instead is to introduce several of its powerful perspectives and, for each, present some important applications. We will treat both exact mathematical analysis and computational techniques. Topics will include:

1. Nondimensionalization and scaling analysis.
   Applications: Kolomogorov's 5/3 scaling in turbulence.
2. Regular asymptotics — mean-field laws, mass-action laws, large number limits for large systems, van Kampen expansions, Kramers-Moyal expansions, Gronwall's estimates
   Applications: short time orbital mechanics.
3. Singular asymptotics — multiscale systems, homogenization, WKB expansions, boundary layers. traveling waves in reaction-diffusion systems, large deviations.
   Applications: semiclassical approximations in quantum mechanics.
4. Complex systems — spectral theory, stability analysis.

Each topic will include an overview of open problems and a list of resources where to look next. The course will be useful for graduate students who are interested in eventually doing research in applied mathematics.

Instructor Vera Mikyoung Hur, verahur@math.uiuc.edu.

Time and location TBA.

Texts 1. G. I. Barenblatt, *Scaling, self-similarity, and intermediate asymptotics.* Cambridge University Press, Cambridge, 1996.
2. M. H. Holmes, *Introduction to the foundations of applied mathematics.* Springer, New York, 2009.
3. M. H. Holmes, *Introduction to perturbation methods.* Springer-Verlag, New York, 1995.
4. C. W. Gardiner, *Handbook of stochastic methods for physics, chemistry and the natural sciences.* Springer-Verlag, Berlin, 2004.
5. Adam Shwartz and Alan Weiss, *Large deviations for performance analysis.* Chapman & Hall, London, 1995.

Grading Homework assignments collected throughout the semester.

Prerequisites The course will require undergraduate background in ODEs, PDEs, probability theory (MATH 441, 442, 461, or equivalents). Graduate courses in the areas can only help. Interested students who are not sure if they have sufficient background are encouraged to email me at verahur@math.uiuc.edu and discuss their readiness.

Math 562

Professor Dey

Course Topics: This is the second half of the basic graduate course in probability theory. The goal of this course is to understand the basic theory of stochastic calculus. We will cover the following topics: (1) Brownian motion; (2) continuous time matingales; (3) Markov processes; (4) stochastic integrals; (5) Ito's formula; (6) representation of martingales; (7) Girsanov theorem and (8) stochastic differential equations. If time allows, we will give a brief introduction to mathematical finance.

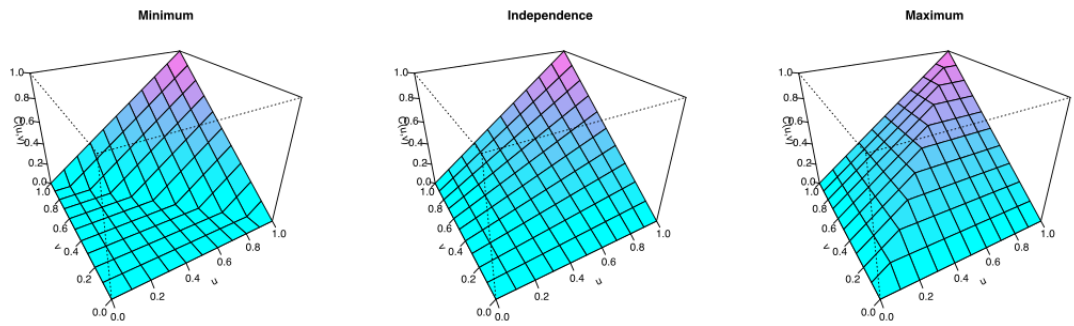
Prerequisite:
1. Math 540 Real Analysis I - we will review measure theory topics as needed.
2. Math 541 is also nice to have, but not necessary.
3. Math 561 Probability Theory I - However, if you have not taken Math 561, but are willing to invest some extra time to pick up the necessary materials from 561, you may register for this course.

Grading: Your grade will depend on homework assignment and a possible final exam or paper presentation.

Text: (Recommended)
1. Jean-Francois Le Gall: Brownian Motion, Martingales, and Stochastic Calculus, Springer, 2016.
2. D. Revuz and M. Yor: Continuous martingales and Brownian motion (3rd edition), Springer, 1999;
3. I. Karatzas and S. E. Shreve: Brownian motion and stochastic calculus (2nd edition), Springer, 1991;

Fall 2018 MATH 563
Risk Modeling and Analysis

Dr. Runhuan Feng, FSA, CERA.



In our complex and interconnected world, dependence modeling has become an increasingly important tool for quantifying, assessing and managing risks in all walks of life. This course provides an introduction to the actuarial theory for modeling dependent risks.

Students will be able to learn various mathematical models and techniques to measure, compare and model dependence. Specific topics include:

- Risk measures;
- Stochastic order relations;
- Bivariate and multivariate copulas;
- Concordance measures;
- Dependence order;
- Integral orderings and probability metrics;
- Stochastic bounds on functions of dependent risks.

Prerequisite: Math 409 or its equivalent. (Students without prerequisite can register with instructor's consent.)

Math 564

Professor DeVille

This is a graduate course on applied stochastic processes, designed for those graduate students who are going to need to use stochastic processes in their research but do not yet have the measure-theoretic background to take the Math 561-562 sequence. Measure theory is not a prerequisite for this course. However, a basic knowledge of probability theory (Math 461) is expected, as well as some knowledge of linear algebra (Math 415/416) and analysis (Math 444/447). The goal of this course is a good understanding of basic stochastic processes and their applications. We will mainly follow parts of "Probability and Random Processes", 3rd Edition by Grimmett and Stirzaker.

Mathematics 570: MATHEMATICAL LOGIC

Fall 2018

Instructor: Lou van den Dries

MWF from .....to ..... in .....

This course gives an introduction to *First Order Logic* (Predicate Logic). No previous study of logic is assumed. Included in the course are:

- The completeness and compactness theorems for first order logic. The first says that provability from a set of axioms is equivalent to validity in all models of the axioms. The second of these is basic to model theory.
- Elements of model theory: theorem of Skolem-Löwenheim, complete theories, back-and-forth, quantifier elimination, Presburger arithmetic.
- Elements of computability theory and Gödel's incompleteness theorem. The latter says that no system of effectively given axioms that includes some basic arithmetic can be complete. Also a discussion of decidable theories with examples.

There will be a midsemester exam, a final exam, and regular homework.

Prerequisites: For undergraduates, Math 414 or consent of the instructor. Some knowledge of (naive) set theory is desirable.

A set of notes by the instructor will be made available.

Math 580

Professor Balogh

Syllabus: This is a rigorous, graduate level introduction to combinatorics. It does not assume prior study, but requires mathematical maturity; it moves at a fast pace. The first half of the course is on enumeration. The second half covers graph theory. There are some topics that are treated more in depth in advanced graduate courses
(Math 581, 582, 583, 584, 585): Ramsey theory, partially ordered sets, the probabilistic method and combinatorial designs (as time permits).

Textbook: The FALL 2018 edition of the text COMBINATORIAL MATHEMATICS
(by Douglas West) will available at TIS Bookstore.

REQUIREMENTS: A raw score of 80% or higher guarantees an A while a score of 60% or higher guarantees a B- (grade drops by 5%). Additionally, for an A, in the final exam minimum 50%, for B+ (passing com) 40% required. (Near) weekly assignments. Each assignment will have 6 problems of your choice of 5/6 are graded.
There are 10 homework assignments, each worth 6%, two tests, each 10%, and a final exam for 20%.
The gradings: 80%− : A, 75%− : A−, 70%− : B+, 65%− : B, etc.
Note that the writings of the solutions must have a high quality and typed, if the argument is messy or not typed then even if the solution is correct it could be returned without grading with 0 points.

Late homework policy: In case the homework is not submitted on time, it could be submitted for the next class, with losing 10% of the score. If there is offical or medical reason then try to notify me in advance via e-mail.
RESOURCES: Electronic mail is a medium for announcements and questions.
PREREQUISITES: There are no official prerequisites, but students need the mathematical maturity and background for graduate-level mathematics.

Course Description — Fall 2018

MATH 583

PARTIALLY ORDERED SETS AND MATROIDS

Instructor: Professor József Balogh, 233B Illini Hall, 244-1918, jobal@math.uiuc.edu.
Office hours: TBA
Web page: http://www.math.uiuc.edu/∼jobal/math583

TEXT: **D. B. West, The Art of Combinatorics, Volume III: Order and Optimization.** The current version will be at TIS Bookstore, 707 South Sixth Street, about $ 35. There also could be some handouts by instructor.

TOPICS: We discuss partially ordered sets and matroids, with applications to measuring preferences, sorting, searching, optimization, etc. Simple examples of *partially ordered sets* (posets) include a family of sets ordered by inclusion or a set of natural numbers ordered by divisibility. Additional topics might include applications of the Szemerédi Regularity Lemma and the hypergraph container method.

Poset structure: Dilworth's Theorem and generalizations, maximum antichains, symmetric chain decompositions, distributive lattices, comparability graphs. Dilworth's Theorem is a min/max relation: the maximum number of pairwise incomparable elements equals the minimum number of chains covering the elements. Lattices are posets having additional special algebraic structure.

Linear extensions: Preference relations, order dimension, geometric representations, correlational inequalities, sorting and searching. Arrow's Impossibility Theorem states that there is no way to aggregate the preferences of voters to satisfy four reasonable axioms. The dimension of a poset is the minimum number of linear criteria that determine it. Representations assign geometric sets to model the poset as an inclusion order. Correlational inequalities study events when the linear extensions are equally likely

Extremal problems: Extremal families of sets, intersecting families, on-line algorithms. What is the maximum size of a family of pairwise-intersecting subsets of an n-set? How to apply modern powerful tools, such as Szemerédi Regularity Lemma and the hypergraph container method.

Matroids: Matroid axiomatics, matroid intersection and union theorems, binary matroids, polymatroids, representations of matroids. Based on hereditary families of subsets of a finite set, matroids generalize notions of independence in graphs and in vector spaces. They yield algorithms to solve generalizations of problems like graph matching and minimum spanning tree. We will emphasize structural aspects of matroids and their variations.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems. The problems require proofs related to or applying results from class. Roughly speaking, 85% of these points suffices for an A, 66% for a B. Discussions between students about problems can help understanding. Collaborations should be acknowledged, and submitted homework should be written (typed) individually. Electronic mail is a good way to ask questions about homework problems or other matters.

PREREQUISITES: Some familiarity with elementary combinatorics. Math 580 is ideal preparation, but any prior contact with partially ordered sets or graph theory may suffice. Similarly, 412 and 413 are sufficient prerequisites.

Math 586
Algebraic combinatorics

Instructor: Alexander Yong

Meeting times: Tuesdays and Thursdays 12:30PM - 1:50PM

Course description: This is a graduate course on methods in algebraic combinatorics. The course will have roughly three components that represent modern preparation in the field. These are enumerative techniques, symmetric functions, and multivariate polynomials.

The intended audience for this course will be PhD students in combinatorics as well as students in representation theory, algebra and geometry where methods from algebraic combinatorics arise. The prerequisite for the course is Math 580 (or instructor approval).

This course will be taught from a combinatorial perspective. The only exceptions (representation theory of the symmetric group and GL_n) will only assume modest background in graduate level algebra.

Grading: Based on homework exercises and a final presentation.

Below I give a list of specific topics and approximate class usage:

I. Generating series (8 hours)

- Ordinary generating series: e.g., tree enumeration, partition enumeration (including the Jacobi Triple Product identity and Euler's pentagonal number theorem), Lagrange inversion
- Exponential generating series: e.g., permutation enumeration, counting connected graphs, theory of species
- Asymptotics of coefficients of generating series

II. Additional general enumerative/algebraic methods (6 hours)

- Möbius inversion (incidence algebra)
- Lindström-Gessel-Viennot lemma
- The Combinatorial Nullstellensatz

III. Symmetric functions (8 hours)

- monomial elementary, homogeneous, power sum bases and their transitions
- Representation theory of the symmetric group, characters and the Murnaghan-Nakayama rule
- Schur polynomials
- Schur polynomials as characters of GL_n representations

IV. Young tableaux: (7 hours)

- The hook-length formula
- RSK correspondence
- The Littlewood-Richardson rule
- Jeu de taquin

V. Polynomials: (9 hours)

- Schubert polynomials
- Symmetric group combinatorics including Bruhat order and reduced words
- Quasisymmetric functions
- Macdonald polynomials and their specializations (Demazure characters, atoms, Hall-Littlewood polynomials)

Textbooks will be

- H. S. Wilf, *Generating Functionology*, available for free download at

  `https://www.math.upenn.edu/~wilf/DownldGF.html`
- R. P. Stanley, *Enumerative combinatorics*, vol. 1, 2nd edition, Cambridge University Press, 1997.
- R. P. Stanley, *Enumerative combinatorics*, vol. 2, Cambridge University Press, 1999.
- W. Fulton, *Young tableaux*, Cambridge University Press, 1997.

Math 595: Factorization algebras and factorization spaces

Fall 2018

Instructor: Emily Cliff.

Course content: Beilinson and Drinfeld introduced the notion of factorization algebras and factorization spaces, geometric incarnations of the notion of a vertex algebra. Vertex algebras have an important role to play in representation theory, especially the representation theory of infinite-dimensional Lie algebras. They are also important in mathematical physics, since they describe the symmetries of two-dimensional conformal field theories.

The study of factorization structures allows us to approach these questions using insights and tools from algebraic geometry. Another advantage of this perspective is that the definitions extend naturally to higher dimensions (as shown by Francis and Gaitsgory), whereas the definition of a higher-dimensional analogue of a vertex algebra is still unknown.

In this course, we will begin by extending our foundations of algebraic geometry to cover the necessary classes of spaces and the (differential-graded) categories of sheaves on these spaces that we will need. Next, we will introduce the notions of factorization spaces, factorization algebras, and chiral algebras, and study the relationships between them.

We can then study properties of factorization algebras/chiral algebras, including commutativity, unversal chiral envelopes, and factorization/chiral homology. We will investigate fundamental examples (for example, the Beilinson–Drinfeld affine Grassmannian, and the factorization algebras associated to it) to see how they fit into different areas of mathematics. There will be some flexibility in the course to address topics of particular interest to the participants.

Prerequisites: Familiarity with the language of schemes and sheaves will be assumed, as will a basic knowledge of category theory (adjunctions, limits, colimits, etc.). Knowledge of D-modules will be helpful, but is not required. Prior familiarity with vertex algebras or with mathematical physics is not at all expected.

Students who are interested in the material but not sure whether they have enough background knowledge are welcome to contact me to discuss the matter.

MATH 595—DEFINABILITY THEORY IN EXPANSIONS OF THE REAL LINE

Instructor: Philipp Hieronymi

Time: MWF 11-11:50AM

Place: TBA

Content. The goal of this topics course is to give an up-to-date overview of the state of the study of definable sets in expansions of the real line. In [1] Miller asked what became the overarching question of this research area: "What might it mean for a first-order expansion of the real line to be tame or well behaved?" Model theorists, real-analytic geometers and more recently number-theorists(!) have focused on the o-minimal setting: an expansion of the real line is o-minimal if every definable set has finitely many connected components. However, there are now many documented examples of expansions of even the real ordered group that define sets with infinitely many connected components, but still can be considered as tame (e.g., the topological closure of every definable set has finitely many connected components). The analysis of such expansions nowadays uses a fascinating combination of model-theoretic, metric-geometric, descriptive set-theoretic and automata-theoretic methods.

In this topics course I plan to give an updated version of the influential, but a bit out-dated survey paper [1]. I will review all basic notions (even o-minimality!) and cover the most important theorems in the area. Throughout the course I will try to state as many open and approachable problems as possible, giving interested students a chance to enter this research area.

Prerequisites. Students should have taken Math 570. Knowledge of Math 571 is helpful, but I will design the course such that knowledge of Math 571 is not necessary.

References

[1] Chris Miller, *Tameness in expansions of the real field*, Logic Colloquium '01, Lect. Notes Log., vol. 20, Assoc. Symbol. Logic, Urbana, IL, 2005, pp. 281–316. MR 2143901

OPEN PROBLEMS IN GROUP THEORY AND TOPOLOGY

Igor Mineyev. Math 595, second half of Fall 2018 (October 22 - December 12), MWF 3pm.
`www.math.uiuc.edu/~mineyev/class/18f/595/`

There is a deep relationship between groups, topological spaces, and metric spaces. Studying one helps studying another. Groups can be viewed as geometric objects, to any topological space one can associate its fundamental group, group presentations lead to cell complexes, metric spaces can be studied using group actions, etc. Geometric group theory is the area of mathematics investigating such relations.

In this course I will concentrate on multiple (and very difficult) open problems that emphasize such relations. If you can solve *any one* of those, turn it to me as homework. Here are open-ended themes that I plan to address in the course.

(1) 1-complexes: Cayley graphs of groups, free groups and their subgorups, Nielsen transformations, the Hanna Neumann conjecture (solved), generalizations of HNC, ...

(2) Free groups and free products: one-relator groups, Magnus' *Freiheitssatz* and its generalizations, the Kervaire conjecture, ...

(3) 2-complexes: group presentations, (highly nontrivial) presentations of the trivial group, the Andrews-Curtis conjecture, the Grigorchuk-Kurchanov conjecture, aspherical 2-complexes, the Whitehead conjecture, the Bestvina-Brady construction, ...

(4) Geometric methods: hyperbolic groups and spaces, the boundary at infinity of a hyperbolic group, Cannon's conjecture, ...

(5) 3-manifolds: the Poincaré conjecture (from a group-theoretic perspective), Stallings' approach ("How not to prove the Poincaré conjecture"), surfaces in 3-manifods, ...

If you are interested in exploring these realms deeper, see the references to books on this class website. This course will only run if sufficiently many students sign up.

MATH 595: INTRODUCTION TO NAKAJIMA QUIVER VARIETIES, FALL 2018

Course Meets: First half of the semester, MWF, time TBA

Instructor: Thomas Nevins (`nevins@illinois.edu`)

Prerequisites: Math 500; Math 510 or Math 511 or some basic knowledge of algebraic varieties.

Course Web Page: `http://www.math.uiuc.edu/~nevins/courses/aut18/m595.html`

Nakajima quiver varieties are a special class of algebraic varieties that play an outsized role in contemporary enumerative geometry, symplectic algebraic geometry, geometric representation theory, and mathematical physics.

This half-semester course will provide an introduction to Nakajima quiver varieties, assuming only very basic background in algebraic geometry. In particular, the construction of Nakajima quiver varieties is very concrete (and based in linear algebra!), and we will develop constructions from deeper within algebraic geometry (some geometric invariant theory and moduli theory) largely from scratch and only as needed.

The course will treat basic definitions and constructions; will go in-depth into various examples connected to resolutions of Kleinian singularities, Hilbert schemes, gauge-theoretic moduli spaces, and more; will explore features of symplectic geometry that are illustrated by quiver varieties; and will develop some of the ways in which quiver varieties appear in the subjects mentioned above (representation theory, enumerative geometry, physics).

I will strive to make the course self-contained in technical detail but I hope that the course will give students a glimpse of the wide mathematical world that quiver varieties inhabit. Accordingly, while there will not be homework, I hope a few students may choose to give presentations about topics that interest them and that involve quiver varieties.

MATH 595 Nilpotence and Periodicity in Stable Homotopy Theory

Second half of Fall 2018
(October 22 - December 12)

Instructor: Vesna Stojanoska

Course description: One of the basic and motivating objects of study in algebraic topology are the stable homotopy groups of spheres, which are the endomorphisms of the unit object in the stable homotopy category. They form a graded commutative ring, $\pi_* S$, of which only $\pi_0 S$ is non-torsion, while every other element is torsion and nilpotent. This is probably the most complicated ring you may ever encounter, as its nature is completely elusive.

There is a filtration, called the chromatic filtration, that in principle helps one understand something structural about $\pi_* S$, and which led Ravenel to make an amazing list of conjectures about the structure of the stable homotopy category in [1]. Even more amazing were the proofs that followed shortly thereafter, by Devinatz, Hopkins, and Smith [2, 3]. These results brought Quillen's work on the relationship between formal group laws and homotopy theory to a whole new level, and have shaped homotopy theory and a number of related areas ever since.

In this course, we will start with a quick introduction of the players involved, largely following the first few chapters of Ravenel's book [4]. Then we will cover some of the work in [2,3] in detail. We will try to sketch the proof of the nilpotence theorem in as much detail as time allows; some input results may have to be black-boxed. Assuming the nilpotence theorem, we will prove the thick subcategory theorem.

Prerequisites: Familiarity with the basics of algebraic topology (eg. MATH 525 and 526), and especially homotopy theory (eg. MATH 527).

Texts include:

[1] Ravenel, Douglas C. *Localization with respect to certain periodic homology theories.* Amer. J. Math. 106 (1984), no. 2, 351-414.

[2] Devinatz, Ethan S.; Hopkins, Michael J.; Smith, Jeffrey H. *Nilpotence and stable homotopy theory. I.* Ann. of Math. (2) 128 (1988), no. 2, 207-241.

[3] Hopkins, Michael J.; Smith, Jeffrey H. *Nilpotence and stable homotopy theory. II.* Ann. of Math. (2) 148 (1998), no. 1, 1-49.

[4] Ravenel, Douglas C. *Nilpotence and periodicity in stable homotopy theory.* Ann. of Math. Stud., 128, Princeton Univ. Press, Princeton, NJ, 1992.

MATH 595 Fall 2018, Tu/Th 11:00-12:20

ALGEBRAIC STRUCTURES FOR MACDONALD THEORY AND INTEGRABILITY PART A AND B

PHILIPPE DI FRANCESCO

This is a 2 part, semester long, graduate topics course. The idea is to review the various algebraic structures underlying the theory of Macdonald operators, whose common eigenfunctions are the celebrated Macdonald polynomials. Most of these structures also underly continuous and integrable systems, i.e. systems with a lot of symmetries, allowing for exact solutions. We will present applications to models of statistical physics, as well as combinatorial problems.

The course is open to graduate students in mathematics or physics. We will review all the basic mathematical or physical material, so no extra knowledge is required. The format of the course is mainly lectures and study of research or expository papers.

We will explore properties of the symmetric group and some families of symmetric orthogonal polynomials, the Hecke algebra and its Temperley-Lieb quotient, the Double Affine Hecke Algebra and its functional representation leading to knot invariants, and the interplay between these algebraic structures and quantum cluster algebras attached to Q-systems, and their consequences for the study of characters of graded tensor products.

Part A

1. Symmetric group

Symmetric group, characters and Linear group Symmetric functions Orthogonal symmetric polynomials: Schur, Hall-Littlewood, Jack, Macdonald Related combinatorial problems

2. Hecke algebra HA/Temperley-Lieb algebra TLA

Integrable lattice models and general solutions of Yang-Baxter equation Ideals of TLA Dixmier trace and scalar product Gram matrix The Meander problem

Part B

3. Double affine Hecke algebra DAHA (type A)

Generators and relations Functional representation Macdonald operators Non-symmetric Macdonald polynomials $SL_2(Z)$ action Refined knot invariants Bosonization/plethysms

4. Quantum toroidal algebra qTorA and elliptic hall algebra EHA

Currents and relations Macdonald currents and level 0 representation Elliptic Hall algebra isomorphism Commutation relations for Macdonald operators q-Whittaker limit: quantum determinants and alternating sign matrices

5. From DAHA to cluster algebra CA: Q-systems

q-whittaker limit, Toda difference equation Quantum Q-system and Macdonald difference operators Graded characters Other types