Intergenerational Risk Sharing Plans: Optimality and Fairness

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Overview

Pension Plans Worldwide

P22

The study covers 22 pension markets in the world (P22). They have pension assets of USD 40,173 bn

P22 markets
Australia
Brazil
Canada
Chile
China
Finland
France
Germany
Hong Kong
India
Ireland
Italy
Japan
Malaysia
Mexico
Netherlands
South Africa
South Korea
Spain
Switzerland
UK
US

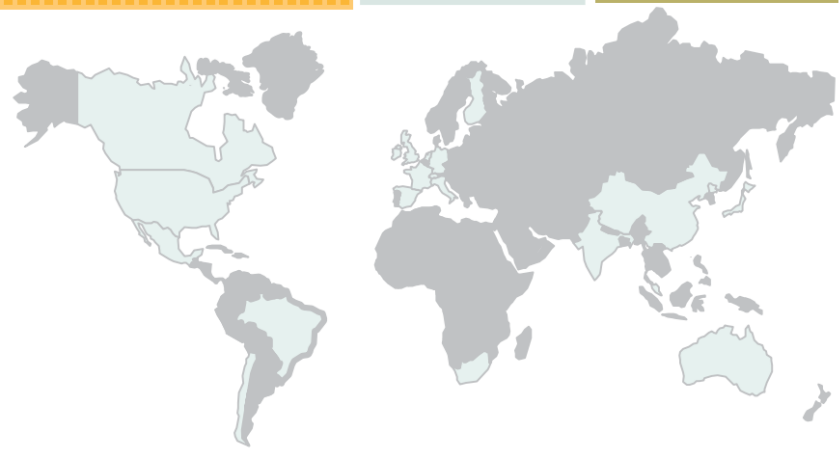
P7

A deeper analysis is performed for the P7, with assets of USD 36,555 bn (91% of P22, 84% of P195)

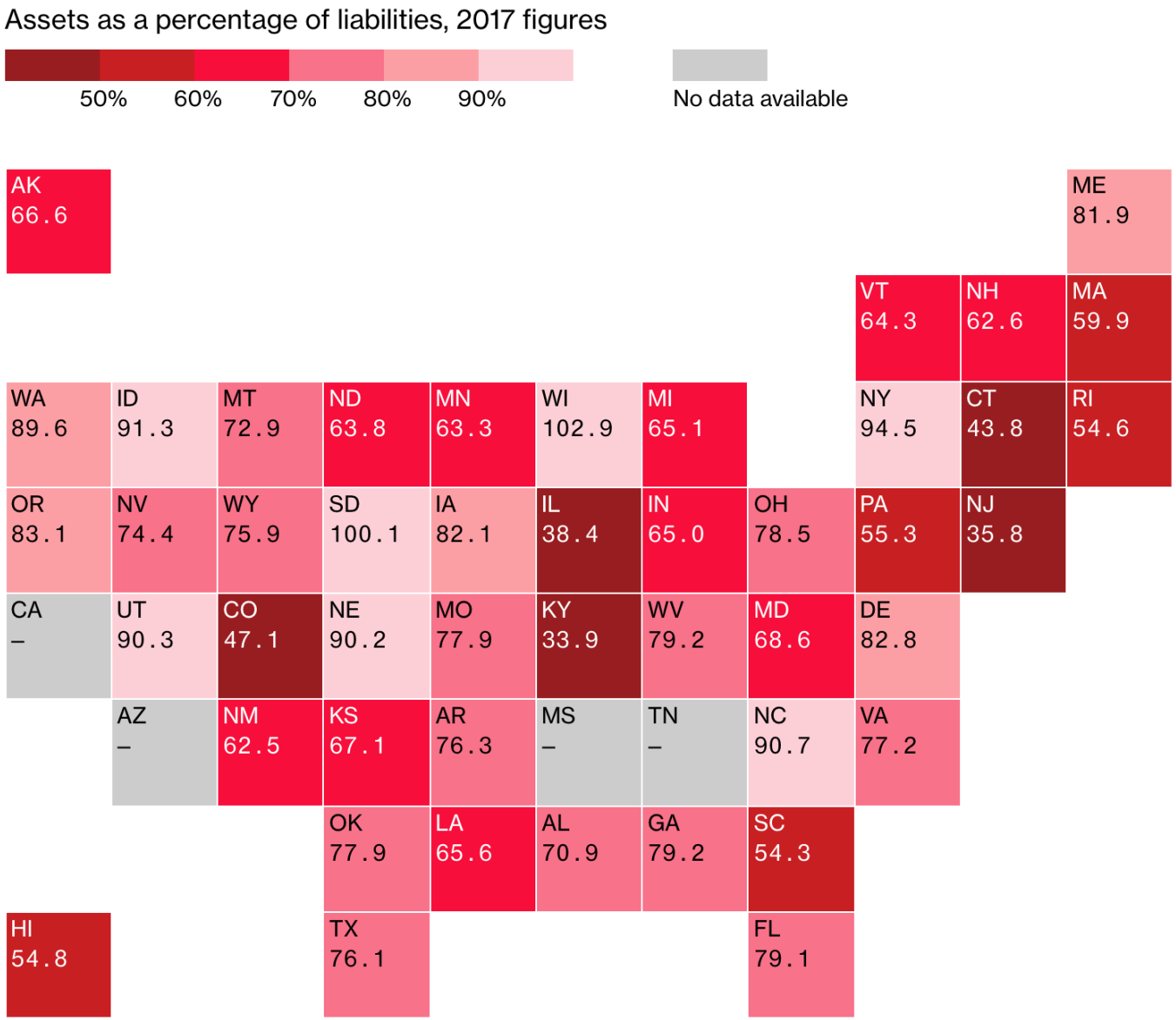
P7 markets
Australia
Canada
Japan
Netherlands
Switzerland
UK
US

P195

Outside the P22 we estimate there is an additional USD 3,000 bn to 4,000 bn of pension assets



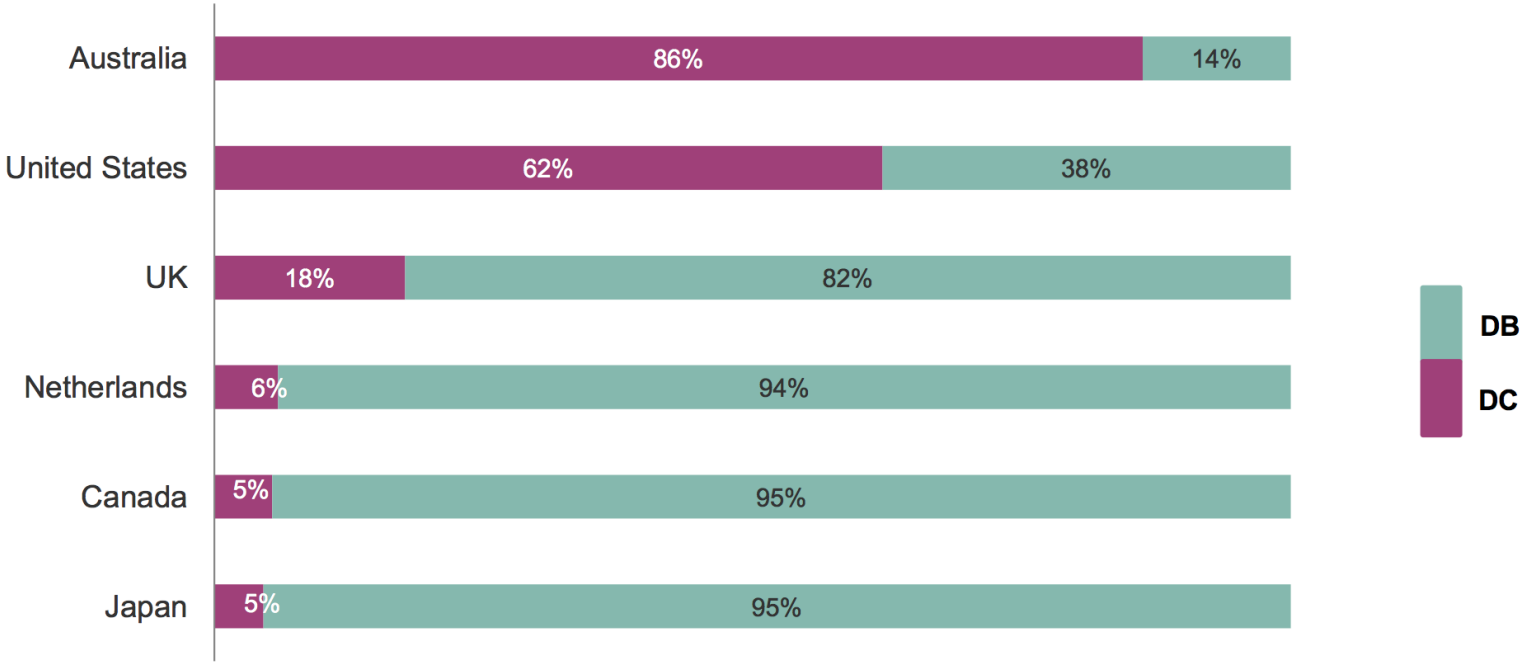
Pension Funding



Traditional Pension Design

- **Defined Benefit (DB) Plan**
  - Provides retirees the "defined" monthly income based on their past salary and years of service.
  - Example: Income = Final Salary × Years in Service × Accrual Rate
  - Sponsors (appear to) bear all risks!
- **Defined Contribution (DC) Plan**
  - Employee makes contributions into a tax-deferred investment account.
  - Employer matches the employee contributions.
  - Employee bears all risks!

Pension Overview



Hybrid Pension Plans

- **Risk-sharing between employees and sponsors**
  - Cash Balance Pension Plan.
  - Defined Benefit Underpin Plan.
  - Second-Election Option.
- Risk-sharing between generations
  - Target Benefit.
  - More Proposed IRS plans: Cui et al. (2011), Wang et al. (2018), Goecke (2013), Bams et al. (2016), Khorasanee (2012), Gollier (2008), Boes and Siegmann (2016), Bovenberg and Mehlkopf (2014), etc.

Cash Balance Pensions

- In U.S., Cash Balance Pensions are regulated as DB plans.
- 12 million participants, with over 1 trillion in assets.
- Each employee has his/her individual account where both employer and employee make contributions ("Pay Credit").
- Account is **notional**, and grows at a pre-specified rate ("Interest Credit").

Interest Crediting Rate	Percentage
Fixed Rate of Return	70.2%
30-year Treasury Rate	23.8%
Actual Rate of Return	6%

Figure: Interest Crediting Rates Chosen

Defined Benefit Underpin Plan

- Also known as the "Floor Offset" plan.
- Main benefit is a DC plan.
- Guaranteed minimum benefit as a DB plan.
- Example: Wilfrid Laurier University.

Investment Account	Minimum Guaranteed Pension
Employer Contribution = 7% Employee Contribution = 9%	1.65% of final salary for every year of service

Second Election Option

- Initial plan can be either DC or DB.
- Before retirement, the employee has option to transfer to DB (DC) plan.
- For DC-to-DB option:
  - The "buy-in" cost for the transfer is the **Accrued Benefit Obligation (ABO)** of the DB plan.
  - The excess of DC balance over the "buy-in" cost will kept in the investment account until retirement.
  - If DC balance is less than the "buy-in" cost, the **employee** is responsible to fund up the difference.
- For DB-to-DC option:
  - The ABO of the DB plan will be the opening investment account balance for the DC plan.
- Example: Florida Public Retirement System.

Hybrid Pension Plans

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Target Benefit

- Also known as "Defined Ambition" .
- Benefit levels are "targeted" rather than "defined" or "guaranteed".
- Example: UBC Staff Pension Plan
  - When Assets are less than Liabilities:

Priority	Action
1.	Reduce the level of future indexing.
2.	Reduce the pension formula.

  - When Assets are greater than Liabilities:

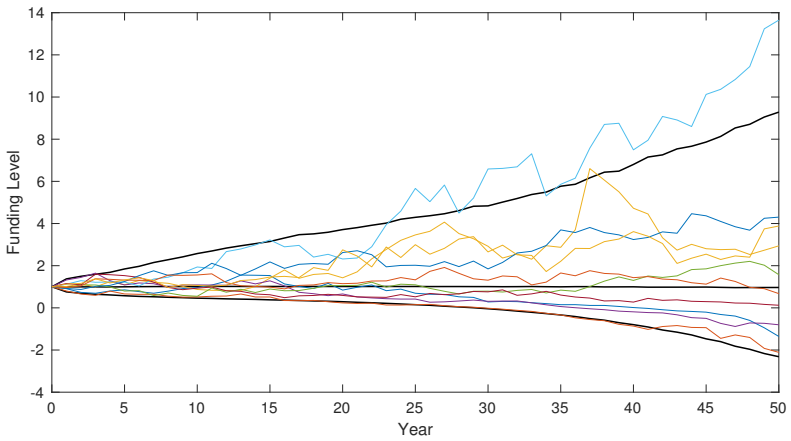
Priority	Action
1.	Reinstate any previous benefit reduction.
2.	Build contingency reserve.
3.	Carry-forward surplus.
4.	Improve pension formula.
5.	Distribute excess funds to members.

Potential IRS Plan Features

- Cash Balance plan with a target fixed rate of return w/o a guaranteed annuity conversion rate.
- The actual interest credit is adjustable according to the funding level.
- A separate smoothing account maybe established.
- Employer makes periodic contributions to the smoothing account as an insurance fee.
- And/or part of return over high watermark contributes to the smoothing account.
- Boes and Siegmann (2016), Bams et al. (2016), Guillén et al. (2006), Goecke (2013), etc.

Funding Level

- Funding level for DB plan



Funding Level

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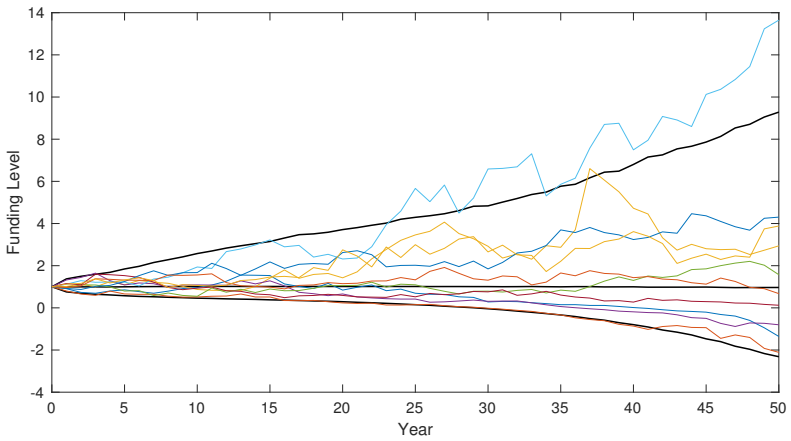
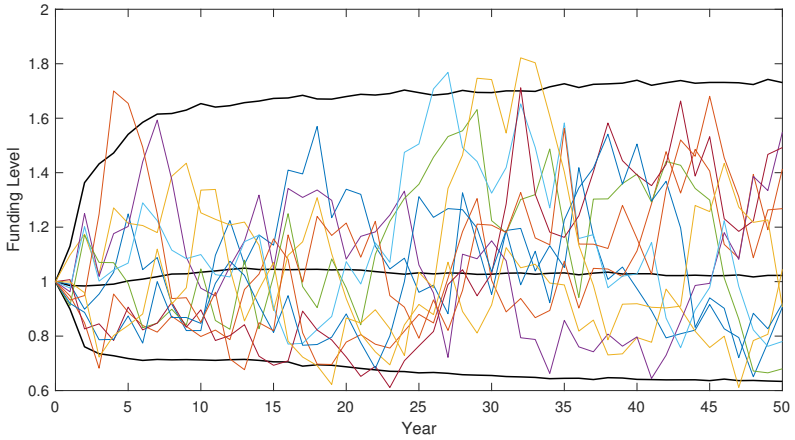


- Funding level for IRS plan



Assumptions and Problem Formulation

- Based on Cui et al. (2011).
- Fixed Retirement Age (R) and Fixed Death Age (N).
- A unit of population for each age. ex. Number of active workers is R.
- Salary is assumed to be 1.
- All contributions are made by the employees.
- The sponsor sets a benchmark contribution level c, and the benchmark benefit b and liability L are calculated through the actuarial equivalence principle:

$$b = \frac{c \times \bar{a}_{\overline{R|}}}{{}_{R|}\bar{a}_{\overline{N-R|}}}$$

$$L = \int_0^N \left(\int_{\max(R,x)}^N b e^{-r(s-x)} ds - \int_{\min(R,x)}^R c e^{-r(s-x)} ds \right) dx$$

Assumptions on Market

- Market consists of a risk-free asset and a risky asset.
- Risk-free asset $S_0(t)$:

$$dS_0(t) = rS_0(t)dt$$

- Risky asset $S_1(t)$:

$$dS_1(t) = \mu S_1(t)dt + \sigma S_1(t)dB_t$$

  where B_t is a standard Brownian Motion.
- Pension asset level X_t follows the SDE:

$$dX_t = \left(X_t(r + \omega_t(\mu - r)) + Rc_t - (N - R)b_t\right) dt + \sigma\omega_t X_t dB_t$$

Optimal IRS Structure

- **Welfare function** - sum of individual utilities $\left(U(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma > 1\right)$.

$$\sup_{\omega_t, b_t, c_t} E\left[\int_0^\infty e^{-\delta t}\left(R\frac{(1-c_t)^{1-\gamma}}{1-\gamma} + (N-R)\frac{b_t^{1-\gamma}}{1-\gamma}\right)dt\right]$$

- **Stability of consumption** - squared distance from target consumption.

$$\inf_{\omega_t, b_t, c_t} \lim_{T\to\infty} \frac{1}{T} E\left[\int_0^T R(1-c_t-c^w)^2 + (N-R)(b_t-c^r)^2 dt\right]$$

where c^w and c^r are the target consumption levels for active employees and retirees.

Theoretical Results

	Welfare Function	Stability of Consumption
Portfolio	$\hat{\omega}_t^W = \frac{\mu - r}{\sigma^2 \gamma}\left(\frac{R}{rX_t} + 1\right)$	$\hat{\omega}_t^S = \frac{\mu - r}{\sigma^2}\left(-1 + \frac{C}{rX_t}\right)$
Contribution	$\hat{c}_t^W = c - \alpha^W\left(\frac{X_t - \psi_w^W L}{R}\right)$	$\hat{c}_t^S = c - \alpha^S\left(\frac{X_t - \psi_w^S L}{R}\right)$
Benefit	$\hat{b}_t^W = b + \beta^W\left(\frac{X_t - \psi_r^W L}{N - R}\right)$	$\hat{b}_t^S = b + \beta^S\left(\frac{X_t - \psi_r^S L}{N - R}\right)$

Optimal IRS Design - Issues

	Welfare Function	Stability of Consumption
α	—	Can be negative
β	—	Can be negative
ψ_w	Can be negative	Can be negative
ψ_r	Can be negative	Can be negative

Optimal IRS design - Linear Risk-Sharing Structure

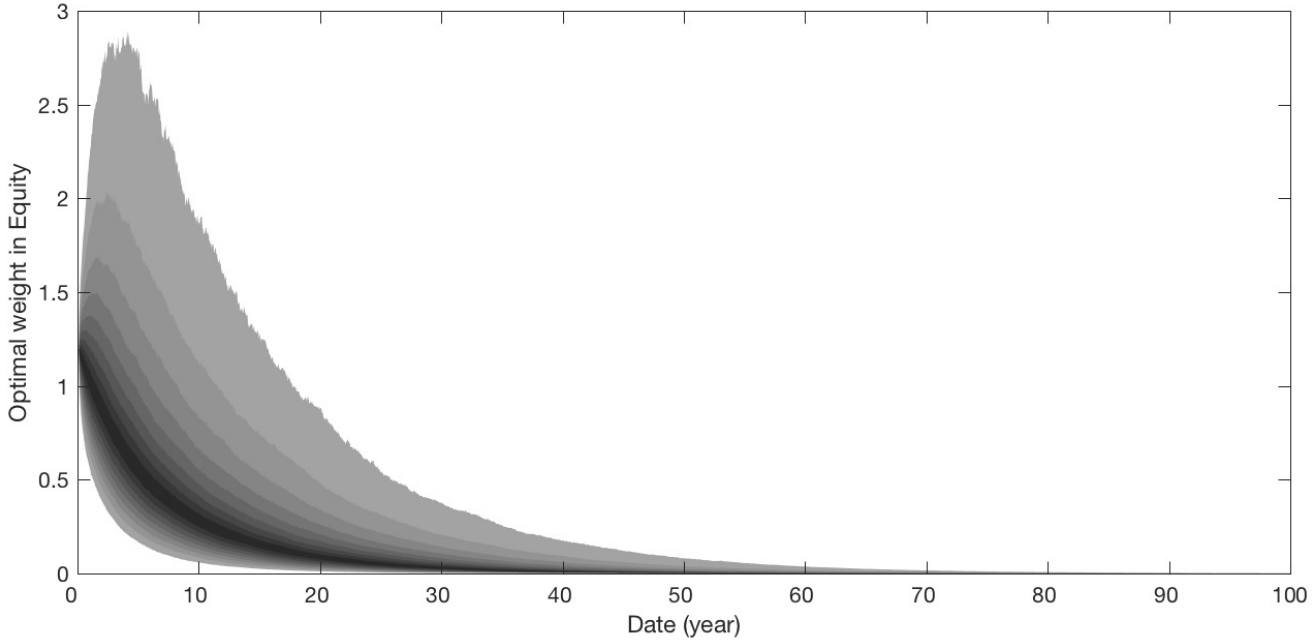
We adopt the linear risk-sharing structure:

$$c_t = c - \alpha \left(\frac{X_t - \psi L}{R} \right)$$
$$b_t = b - \beta \left(\frac{X_t - \psi L}{N - R} \right)$$

The objective function is

$$\inf_{\alpha,\beta,c} \left(\inf_{\omega_t} \lim_{T \to \infty} \frac{1}{T} \left[\int_0^T R(1 - c_t - c^w)^2 + (N - R)(b_t - c^r)^2 dt \right] \right)$$

Optimal Portfolio Weight - Unconstrained Case



Possible Solution

1. Social Planner Discounting ($e^{-\delta t}$).

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4. Constraints

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4. Constraints
   - Constraint on control variable ($l \leq \omega_t \leq u$).

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     Selecting $l > 0$

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   - Constraint on control variable ($l \leq \omega_t \leq u$).
     Selecting $l > 0$
   - Constraint on IRS structure.

Regulation on Recovery Period

- When deficits occur, regulations often require the pension fund to create a recovery plan.
- Funding level will be automatically recovered in an IRS plan.
- Here we define the recovery period for our IRS as:

$$t^* = \inf\left\{ t : t > 0, \frac{X_t}{L} = f_r \,\middle|\, \frac{X_0}{L} < f_r \right\}$$
$$dX_t = (rX_t - N(1-\Omega)c_t + N\Omega b_t)\, dt$$

where all pension assets are invested in the risk-free bond.

Regulation on Recovery Period

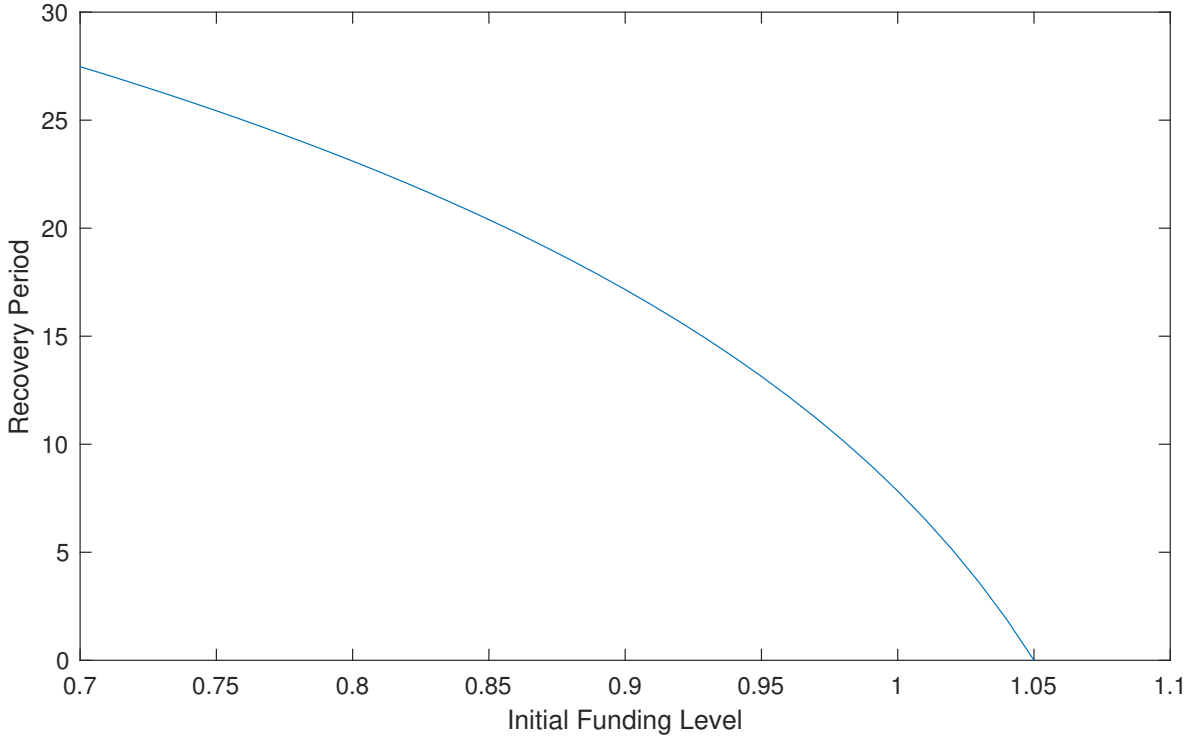


Figure: The recovery period when $\alpha = 0.02$, $\beta = 0.06$

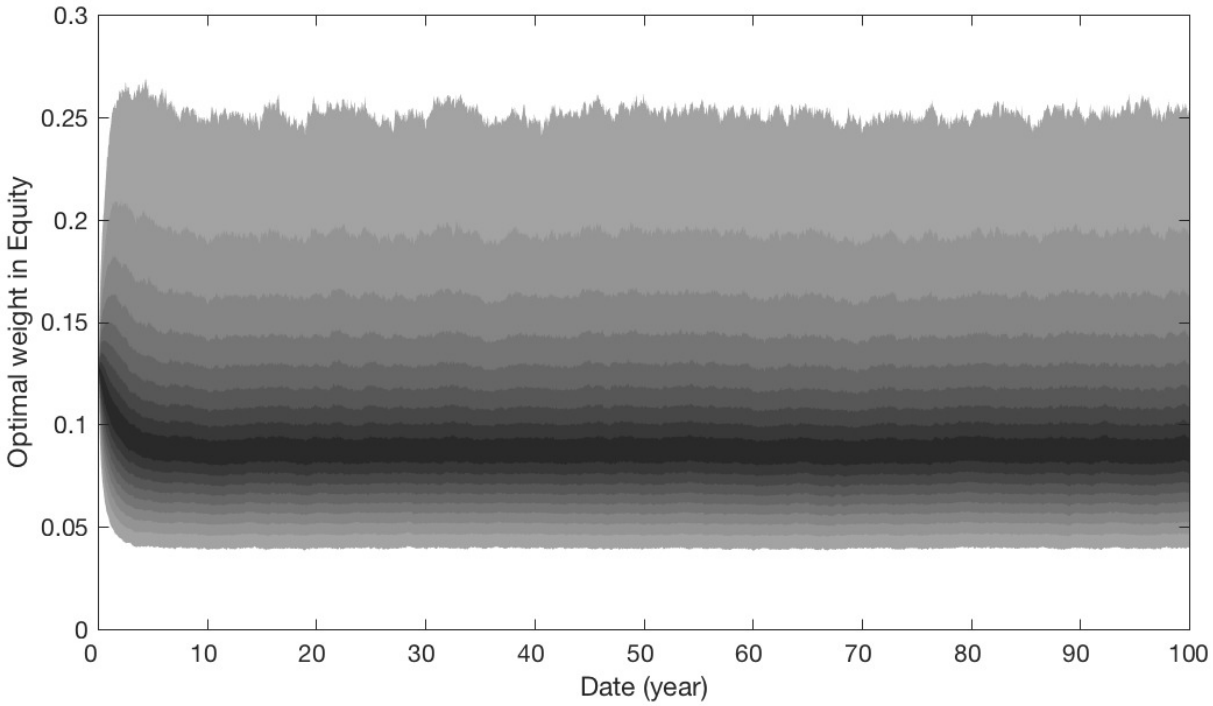
Regulation on Recovery Period

- The constraint can be expressed as $t^*(\xi)$, where $\xi = \alpha + \beta$, and we are trying to find ξ^* such that $t^*(\xi^*) - t_r = 0$, with t_r being the required recovery time.
- To ensure the uniqueness of solution, we define ξ^* as

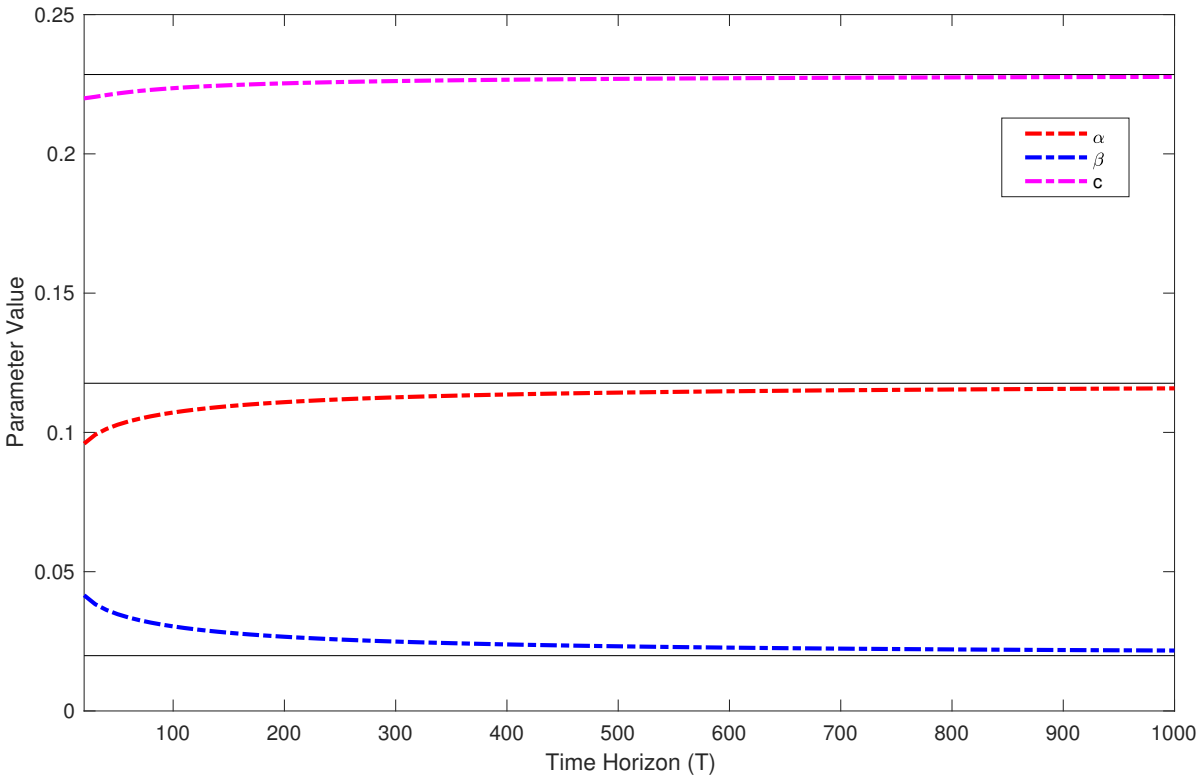
$$\xi^* = \max\left\{\xi' \middle| t^*(\xi') = t_r, (t^*(\xi) < t_r, \forall \xi > \xi')\right\}$$

- The constrained parameter domain is simply $\left\{(\alpha, \beta) \middle| \xi^* \leq \alpha + \beta \leq 1, \alpha \geq 0, \beta \geq 0\right\}$.

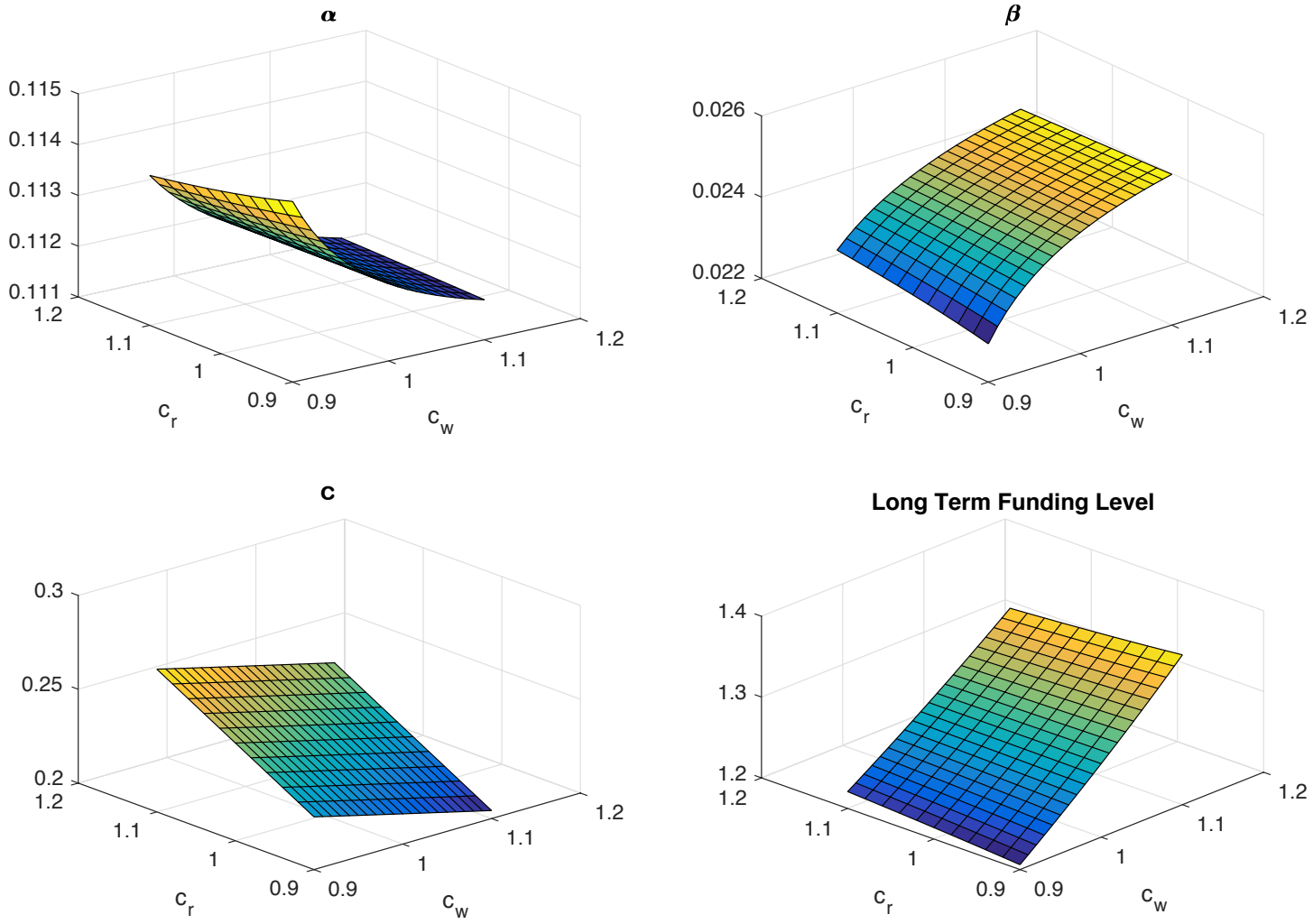
Optimal Portfolio Weight - Constrained Case



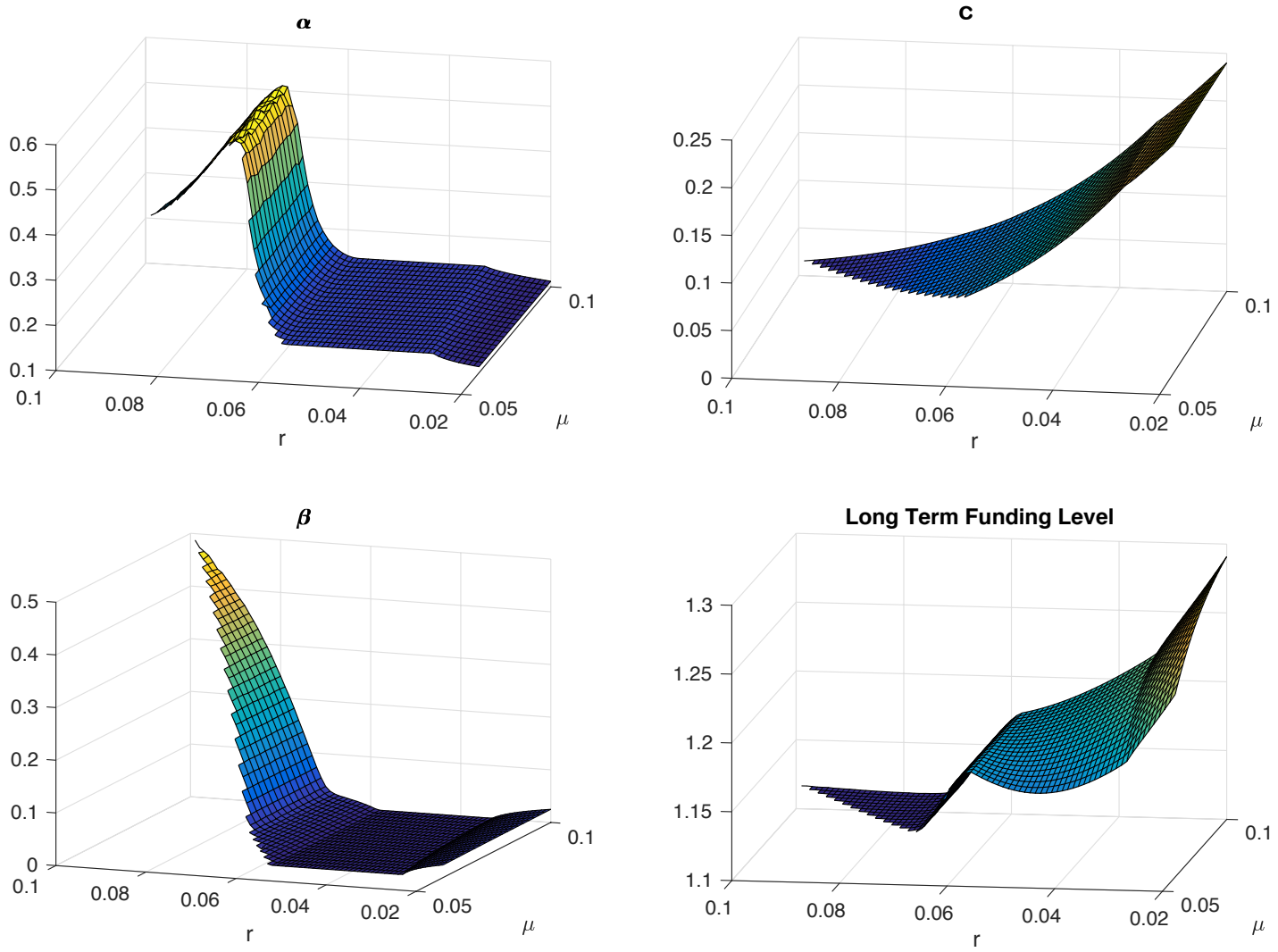
Convergence in Time ($T \to \infty$)



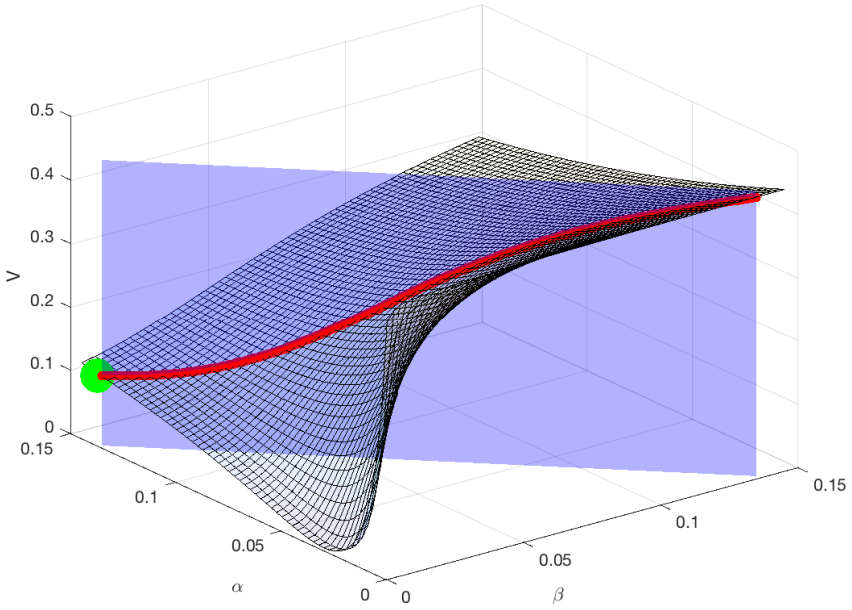
Sensitivity Tests for different c^r and c^w



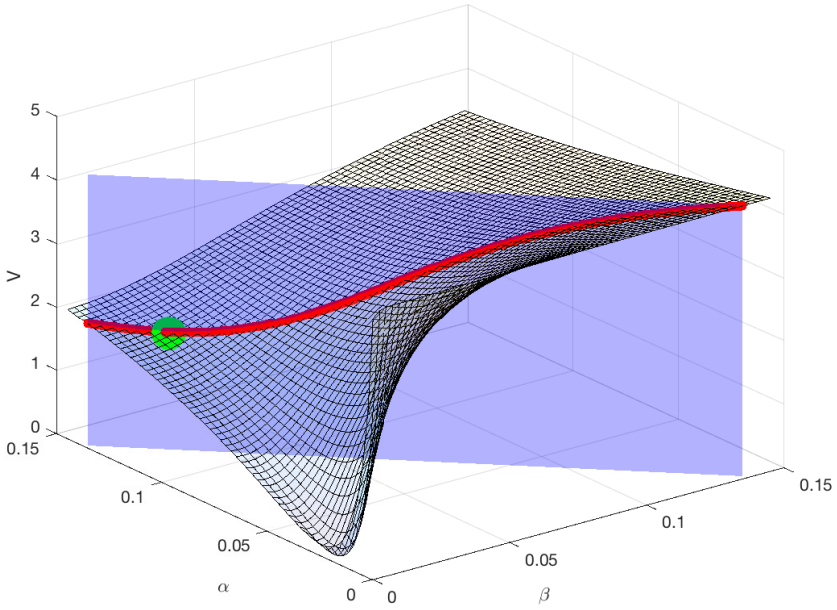
Sensitivity Tests for different r and μ



Optimal Values in the Constrained Case



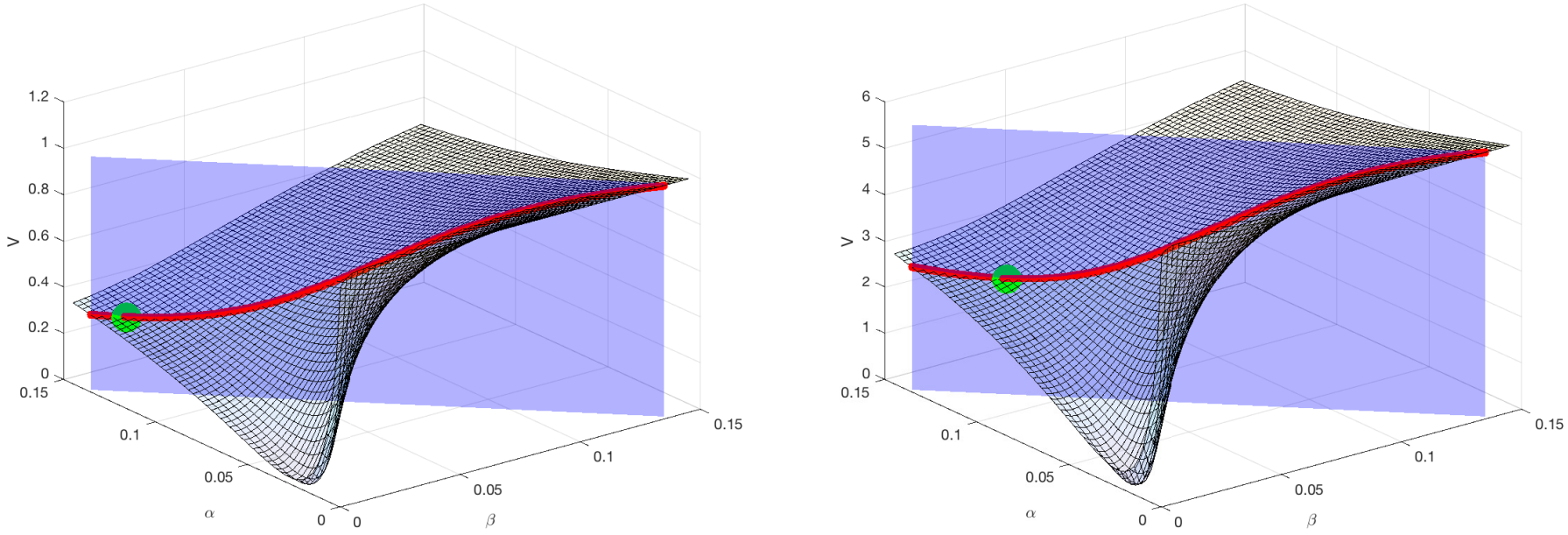
(a) $c^w = 0.9,\ c^r = 0.9,\ r = 0.025$



(b) $c^w = 1.1,\ c^r = 1.1,\ r = 0.025$

Figure: The value function under different α and β with optimal c. The red line is the intersection between the regulatory constraint (blue plane) and the value function surface.

Optimal Values in the Constrained Case: Demographic Shift



(a) $c^w = 0.9,\ c^r = 0.9,\ r = 0.025$ (b) $c^w = 1.1,\ c^r = 1.1,\ r = 0.025$

Figure: The value function under different α and β with optimal c, when the percentage of retirees is $\Omega = 0.4$. The red line is the intersection between the regulatory constraint (blue plane) and the value function surface.

Conclusions and Future Work

Conclusions

- We provide theoretical justification of the linear risk-sharing structure.
- We illustrate the necessity of incorporating regulatory constraints.

Future Work

- Modelling changes in population structure.
- Incorporate constraints directly in optimal control problem.
- The responsibilities of the sponsor.
- More realistic modelling assumptions.

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