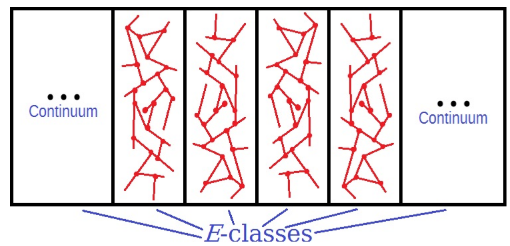
COUNTABLE BOREL EQUIVALENCE RELATIONS

MATH 595 | Anush Tserunyan | 2019 SPRING

Location TBA | TueThu 11:00–12:20

An equivalence relation E on a Polish space X (e.g., $X := \mathbb{R}, L^1(\mathbb{R}), \mathbb{N}^{\mathbb{N}}$) is *Borel* if it is a Borel subset of X^2, and it is *countable* if each E-class is countable. Countable Borel equivalence relations (CBERs) naturally arise as orbit equivalence relations of Borel actions of countable groups (e.g., $\mathbb{Q} \curvearrowright \mathbb{R}$ by translation). From another, rather combinatorial, angle, a CBER E can always be viewed as the connectedness relation of a locally countable Borel graph (e.g., take E as the set of edges). These connections between equivalence relations, group actions, and graphs create an extremely fruitful interplay between descriptive set theory, ergodic theory (probability measure preserving actions), measured group theory, probability, descriptive graph combinatorics (Borel colorings and matchings), and geometric group theory (trees and quasi-isometry).



This course will feature this interplay. On one hand, we will learn some tools from each of the aforementioned subjects to analyze the structure of CBERs. On the other hand, we will utilize the basic theory of CBERs to prove some well-known results in those subjects.

Prerequisites: Being comfortable with

- basic *soft analysis* (basic pointset topology and metric spaces),
- basic *measure theory* (Borel and measurable sets and functions),
- *group actions* (definition, orbits),
- and *graph-theoretic terminology* (cycles, connectedness, trees).

No knowledge of descriptive set theory is required, we will cover what we need on the fly.

Coursework: Occasional homework, as well as one in-class presentations of a relevant topic/paper.

References: We will use parts of

- "The Theory of Countable Borel Equivalence Relations" by A. Kechris [pdf [1]],
- "Topics in Orbit Equivalence" by A. Kechris and B. Miller [Springer link [2]],
- and "Introduction to Descriptive Set Theory" by A. Tserunyan [pdf [3]].

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[1] Online at http://www.math.caltech.edu/~kechris/papers/lectures%20on%20CBER01.pdf

[2] Free for UIUC to download at https://link.springer.com/book/10.1007/b99421

[3] Online at https://faculty.math.illinois.edu/~anush/Teaching_notes/dst_lectures.pdf