Math 595 KM -- Spring 2019

GEOMETRIC REPRESENTATION THEORY, ENUMERATIVE GEOMETRY AND QUIVERS.

KEVIN MCGERTY

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Since their introduction in the seminal paper [N1], Nakajima's quiver varieties have played a central role in geometric representations theory. They provide a rich class of examples of smooth holomorphic symplectic varieties (indeed hyper-Kähler manifolds) and thus are an important testing-ground for the emerging field of "symplectic representation theory". This course will review some recent developments in this direction, including categorification of Nakajima's cohomological constructions of representations of Lie algebras, and enumerative topics such as the calculation of the quantum D-module for quiver varieties following the work of Maulik and Okounkov [MN]. The list below gives an outline of topics to be covered, which is probably over-ambitious, and the focus of the course will be influenced by the interests of the audience.

Topics:

(1) Review of quiver varieties: construction, Poisson geometry etc.
(2) Quantization of quiver varieties via quantum Hamiltonian reduction, and brief discussion of categories of sheaves of modules for such quantizations.
(3) Categorification of Nakajima's actions on cohomology of quiver varieties.
(4) Centre constructions and ring structure on cohomology of moduli space of framed instantons over the affine plane.
(5) Review of quantum cohomology
(6) R-matrices and Yangians
(7) Maulik-Okounkov: stable envelopes and calculation of quantum cohomology of quiver varieties via Yangians.
(8) Cohomological Hall algebras and relation to the Maulik-Okounkov Yangian.

REFERENCES

[N1] H. Nakajima; *Quiver varieties and Kac-Moody algebras.* Duke Math. J. 91 (1998), no. 3, 515–560.
[MN] D. Maulik,; A. Okounkov; *Quantum Groups and Quantum Cohomology*, arXiv:1210:1287.
[SVV] Shan, P.; Varagnolo, M.; Vasserot, E. On the center of quiver Hecke algebras. Duke Math. J. 166 (2017), no. 6, 1005–1101.

MATHEMATICAL INSTITUTE, OXFORD AND UNIVERSITY OF ILLINOIS.

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