Course Meets: First half of the semester, day/time TBA

Instructor: Thomas Nevins (nevins@illinois.edu)

Prerequisites: Math 500; Math 510 or Math 511 or some basic knowledge of algebraic varieties.

Course Web Page: http://www.math.uiuc.edu/~nevins/courses/aut20/m595p-adic.html

$p$-adic integration was developed as an answer to the question of how to integrate differential forms on manifolds over the field $\mathbb{Q}_p$. This course will provide an economical introduction to the theory with selected cohomological applications.

There are multiple versions of “$p$-adic integrals.” This course will focus on the one developed in works by Tate, Serre, etc. and used in the 1990s by Batyrev to prove that birational, smooth projective Calabi-Yau varieties have the same Betti numbers. The course will not focus on maximal generality, and will not assume any familiarity with $p$-adic numbers or real analysis beyond Math 540. Instead, its goal is to provide students with basic tools that can then be used to understand various applications in algebraic geometry, number theory, and representation theory.

A principal goal of the course will be to develop enough about $p$-adic integrals to prove Batyrev’s theorem on $K$-invariance of Betti numbers. An added benefit is that the course should prepare interested students, postdocs, and faculty to participate in a working seminar to understand more recent work of Groechenig-Wyss-Ziegler on mirror symmetry for Hitchin systems and Ngô’s “Geometric stabilization theorem.”

The course will rely on freely available sources as its supporting texts.