Course Descriptions
Spring 2020

Department of Mathematics
University of Illinois

Math 500

Prof Deville

This is the standard graduate-level introductory course, and we will follow "Abstract Algebra" by Dummit and Foote. The four main components in the course are [1] Group Theory, [2] Ring Theory, [3] Modules and Vector Spaces, and [4] Fields and Galois Theory. We will spend roughly the same amount of time on each component.

INTRODUCTION TO GEOMETRIC GROUP THEORY

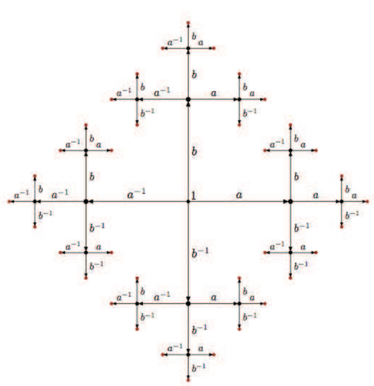
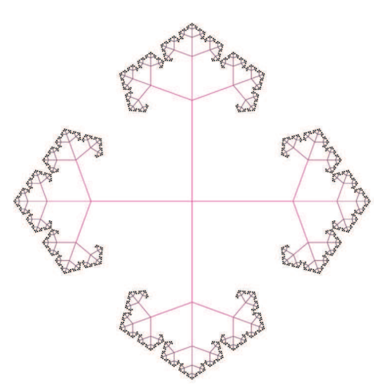
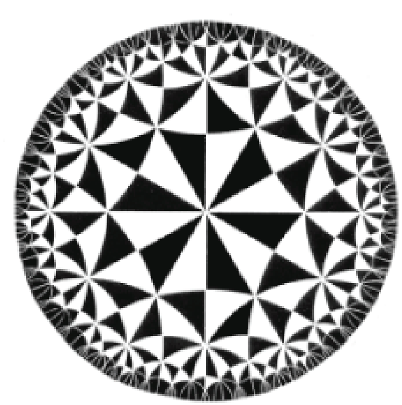
Igor Mineyev. Math 503, Spring 2020.
`https://faculty.math.illinois.edu/~mineyev/class/20s/503/`

Geometric group theory is not a subject in itself; it is rather the place where various areas of mathematics interact: algebra, topology, geometry, analysis, computational methods, and more. Here is the tentative list of topics that I intend to cover in this course; this might be modified somewhat as we proceed.

- Cayley graphs, the word metric, groups as metric spaces, quasiisometry.
- Free groups and their subgroups, their descriptions via Stallings' graphs, Schreier's subgroup theorem, Nielsen transformations, automorphisms of free groups.
- Group actions on trees, free products, ping-pong lemma, free products with amalgamations, HNN-extensions, graphs of groups.
- Groups presentations by generators and relators, van Kampen diagrams, van Kampen theorem, isoperimetric function, algorithmic problems in group theory.
- Examples of quasiisometry invariants: growth of finitely generated groups, ends, isoperimetric functions, amenability, solvability of the word problem, asymptotic cones, hyperbolicity.
- Word hyperbolic groups and spaces, their numerous definitions and properties, examples, the ideal boundary, quasiconformal and conformal structures on the ideal boundary.

There is no required textbook; the following sources might be helpful, among many other.

- Magnus, Karras, Solitar. Combinatorial group theory.
- Lyndon, Schupp. Combinatorial group theory.
- Jean-Pierre Serre. Trees.
- Ghys, Haefliger, Verjovsky. Group theory from a geometrical viewpoint.
- Collins, Grigorchuk, Kurchanov, Zieschang. Combinatorial group theory and applications to geometry.
- John Meier. Groups, graphs and trees.
- Gilbert Baumslag. Topics in combinatorial group theory.

Math 519
Differentiable Manifolds II

Instructor: Ely Kerman

Lectures: Tuesday–Thursday, from 11:00 to 12:20 in 447 Altgeld Hall

Course Description. In this second course on the theory of smooth manifolds we will study a variety of rich structures on manifolds including vector bundles, principle bundles, Riemannian metrics, covariant derivatives, connections and curvature. These structures will lead us to several theorems in which analysis is used to reveal deep connections between the geometry and topology of manifolds. These will include de Rham theory, the theory of characteristic classes, Chern-Weil theory, and the Gauss-Bonnet-Chern theorem.

Grading: Grades will be based on problem sets.

Mathematics 525 — Topology

Instructor: Charles Rezk
Office: 242 Illini Hall
Phone: 5-6309
Email: rezk@math.uiuc.edu
Webpage: http://www.math.uiuc.edu/~rezk/

Course requirements:
Homework: There will be weekly homework assignments (generally due each Monday).

Tests: There will be one in class midterm, and an in-class final.

Prerequisites:
Some basic algebra (especially groups and abelian groups), such as is covered in 417 or 500. Some point set topology is recommended; in particular, it helps if you've seen the notion of a general topological space (as covered in 432 and/or 535); however, we'll review everything we need, so it's not essential that you've seen it before.

Texts: The primary text will be:

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001. This book is also available for free at
  http://www.math.cornell.edu/~hatcher/

This will be supplemented with additional course notes.

Course schedule: This course is an introduction to the basic concepts of algebraic topology, namely: the fundamental group, covering spaces, and homology.
Topics will include:

- The fundamental group.
- Covering spaces and their classification.
- Singular homology.
- The Eilenberg-Steenrod axioms for homology.
- Applications: Brouwer fixed point theorem, fundamental theorem of algebra, Borsuk-Ulam theorem, etc.

Evaluation: This will be based on

- weekly homework assignments (40%),
- one take home midterm (20%),
- an in class final (40%).

MATH 530 COURSE DESCRIPTION

This will be a first course in algebraic number theory, in which we will study the arithmetic of the rings of integers in finite field extensions of $\mathbb{Q}$. This is a subject that is both classical and modern, and in this course we will develop the foundations while also giving vistas of modern questions and developments. Concrete topics we will discuss include:

- integral closure and Dedekind domains;
- unique factorization of ideals and the class group;
- Minkowski theory, the finiteness of the class group, and Dirichlets Unit Theorem;
- valuations and p-adic fields;
- ramification and Galois theory.

Time permitting, we will also dicsuss a selection of topics among the following:

- localglobal principles for Diophantine equations;
- class field theory;
- Hecke and Artin L-functions;
- special value formulas for Dedekind ζ-functions.

Prerequisite: Math 500.

Grading: Grades will be determined based on assigned homework, midterms, and a final exam.

References: There is no required text. The following references may be useful:

- Marcus, Number Fields. E-book available through the UIUC library.
- Milne, Algebraic Number Theory.
- Neukirch, Algebraic Number Theory. E-book available through the UIUC library.

Graduate Course Description
Spring 2020
Math 532: Sieve Methods

Instructor: Kevin Ford

Time/place: MWF 12–12:50; Altgeld 445

Prerequisites: Basic analytic number theory will be very helpful (equivalent of the first half of Math 531; elementary prime number estimates and multiplicative functions, knowledge of the prime number theorem). Some knowledge of basic probability will be helpful but not necessary.

Recommended Texts: *Sieve methods*, H. Halberstam and H.-E. Richert, Dover paperback edition (highly recommended for purchase).
Opera de Cribro, J. Friedlander and H. Iwaniec, AMS, 2010.

Course Description.
A *sieve* is a tool for bounding the number of integers from a finite set which remain when those divisible by a set of primes are excluded. Several different sieves will be covered in this course, each tailored to certain types of applications. We will take a probabilistic viewpoint, interpreting various results in terms of certain random variables, and discussing the role of independence vs. dependence. Here are some sieves and applications we will learn about:

- The "small sieve", and applications to prime/almost prime values of polynomials, k-tuples of primes, primes in polynomials (The Brun-Titchmarsh inequality), primes in short intervals.
- The "large sieve" and applications to the least quadratic nonresidue modulo a prime, and to the distribution of primes in progressions (Bombieri-Vinogradov theorem)
- Sieving for integers lacking large prime factors ("smooth" numbers), application to large gaps between primes
- prime factors of shifted primes $p \pm 1$, application to the distribution of values of Euler's function
- The Kubilius model of integers, application to integers with a prescribed number of prime factors, the Erdős-Kac theorem
- The Maynard-Tao sieve with applications to bounded gaps between primes and to large gaps between primes.
- The random sieve, connection to large gaps between primes and the Hardy-Littlewood prime k-tuples conjectures.

Grades. The course grade will depend on homework assignments, which will be given periodically.

Math 541

Pavlos Motakis

Math 541. Functional Analysis Instructor Syllabus

1. Review of abstract measure theory.

2. Basic topics on Banach spaces, linear and bounded maps on Banach spaces, open mapping theorem, closed graph theorem. Examples and connection to measure theory.

3. Hahn-Banach theorem, Extreme points, Krein-Milman and Caratheodory. Examples.

4. Compact operators, spectrum and spectral theorem for compact operators on Hilbert spaces.

Further topics: Fredholm alternative, unbounded operators, Riesz representation theorem, Haar measure for locally compact groups, non-linear functional analysis, distributions and Sobolev spaces.

Recommended textbooks:

1. J.B. Conway, A Course in Functional Analysis.
2. W. Rudin, Functional Analysis.
3. G.B. Folland, Real Analysis. Modern Techniques and their Applications.
4. Y. Benyamini and J. Lindenstrauss: Geometric Nonlinear Functional Analysis.

COURSE DESCRIPTION FOR MATH 545

XIAOCHUN LI

This course is an introduction to modern harmonic analysis. The following classic topics are planed to be covered.

- Marcinkiewicz interpolation; Approximation to the identity; L^p theory of Fourier transforms;
- The theory of Calderön-Zygmund singular integrals;
- Littlewood-Paley theory (continuous version and discrete version); Multiplier;
- BMO and Carleson measure; T1 theorem;
- $L^p (p \neq 2)$ unboundedness of the disk multiplier;
- Oscillatory integrals

Lectures: TR 11:00–12:20 in Altgeld.

References:There is NO textbook for the course. The following references are used.

1. Javier Duoandikoetxea, *Fourier analysis*, Graduate studies in math., Vol 29. AMS, 2001.
2. E. M. Stein, *Singular Integrals and Differentiability Properties of Functions* , Princeton Univ. Press, Princeton, 1970
3. E. M. Stein, *Harmonic analysis: Real variable methods*, Princeton Univ. Press, Princeton, 1993
4. E. M. Stein and G. Weiss, *Introduction to Fourier Analysis in Euclidean Spaces*, Princeton Univ. Press, Princeton, 1971

Grading: Homework (100%)

Prerequisites: Solid knowledge of real analysis.

Date: September 29, 2010.

Math 561

Instructor: Renming Song

Office: 338 Illini Hall

Phone (217) 244 6604

Email: rsong@illinois.edu

Homepage: http://www.math.uiuc.edu/~rsong

Test: R. Durrett: *Probability: Theory and Examples (4th edition)* Cambridge University Press, 2010

Course topics: This is the first half of the basic graduate course in probability theory.

The goal of this course is to understand the basic tools and language of modern probability theory. We will start with the basic concepts of probability theory: random variables, distributions, expectations, variances, independence and convergence of random variables.

Then we will cover the following topics: (1) the basic limit theorems (the law of large numbers, the central limit theorem and the large deviation principle); (2) martingales and their applications.

If time allows, we will give a brief introduction to Brownian motion. The prerequisite for

Math 561 is Math 540.

Grading Policy: 40% of your grade will depend on homework assignment, 30% will depend on the midterm test and 30% on the final exam.

MATH 595: Algebraic and Differential Topology in Data Analysis (ADTDA)

Yuliy Baryshnikov

The course will cover some recent applications of topology and differential geometry in data analysis. Tools of differential and algebraic topology are starting to impact the area of data sciences, where the mathematical apparatus thus far was dominated by the ideas from statistical learning, computational linear algebra and high-dimensional normed space theories.

While the research community around ADT topics in data analysis is lively and fast-growing, the area is somewhat sparsely represented in campus syllabi. I wanted to test ground whether there is a constituency for a course covering some of it (with the choice of topics, of course, tilted towards what I find interesting).

Syllabus

1. **Tools from algebraic topology**
   Homotopy equivalence, Simplicial homology, Nerve lemma, Dowker's theorem
   Applications: Sketches: Merge trees; Reeb graphs. Netflix problem complexes
2. **Tools from differential topology**
   Useful topological spaces: manifolds, subanalytic sets, simplicial complexes. Transversality, Sard's theorem. Whitney's embedding theorem.
   Applications: Dimensionality reduction; Embeddings
3. **Topological Approximations**
   Vietoris-Rips and Čech complexes. Topology reconstruction from dense samples: Hausmann-Latscher; Topology reconstruction from random samples: Niyogi-Smale-Weinberger
   Applications: Cech complexes and their topology in robotics and neurophysiology.
4. **Euler calculus**
   Set algebras and valuations; Hadwiger and McMullen's theorems. Integration with respect to Euler characteristics. Fundamental Kinematic Formula.
   Applications: Gaussianity tests for random fields; Topological Sensor Networks
5. **Topological Inference**
   Persistent Homology: Algorithms; Stability. Biparametric persistence
   Applications: Image patches spaces. Textures and characterization of materials
6. **Aggregation**
   Spaces with averagings. Arrow theorem and Topological Social Choice. Aggregation in CAT(0) spaces
   Applications: Consensus in phylogenetic analysis. Political polarization
7. **Clustering**
   Basic clustering tools. Kleinberg's Impossibility theorem. Carlsson-Memoli functorial approach to clustering.

The course will rely mainly on the recent papers, and a few textbooks, like
R Ghrist, Elementary Applied Topology, Createspace, 2014
A Hatcher, Algebraic Topology, CUP, 2002
V Prasolov, Elements of Combinatorial and Differential Topology, AMS, 2006

To receive credit the students will be expected to take (a fraction of) class notes, and either present a paper (from a list), or to run a computational project (from a set of provided topics).

COURSE: INFINITE FAMILIES IN STABLE HOMOTOPY GROUPS

MATH 595 (INSTRUCTOR: D. CULVER)

The main goal of this course will be to explain the so-called chromatic perspective to stable homotopy theory from a computation lens. The main goal of this perspective is to find infinite families in the stable homotopy groups of spheres and arises from a very interesting connection between topology and one-dimensional formal group laws. The main object of study for this class will be the Adams-Novikov spectral sequence

$$E_2^{s,t} = \mathrm{Ext}^{s,t}_{MU_*MU}(MU_*, MU_*) \Longrightarrow \pi_{t-s}S.$$

Here MU denotes the cohomology theory arising from complex cobordism. The starting point of chromatic homotopy theory comes from the observation that the E_2-term is, in fact, the cohomology of the moduli stack of one-dimensional commutative formal groups. It turns out that this observation leads to a very systematic approach to computing stable homotopy and has led to a great deal of concrete calculations.

Here is a tentative list of topics I hope to cover.

(1) Review of the Adams spectral sequence and some concrete calculations examples.
(2) Overview of Adams' calculation of the image of the J-homomorphism.
(3) Review of complex oriented cohomology theories and Quillen's theorem.
(4) p-typical formal groups, the Brown-Peterson spectrum, and the Adams-Novikov spectral sequence. Some concrete computations.
(5) Construction and computation of the algebraic chromatic spectral sequence. Calculation of the α, β, and γ-families.
(6) Morava change-of-rings, and the cohomology of Morava stabilizer algebras.

Time permitting, I also hope to cover the following topics.

(1) Ravenel's solution to the nonexistence of odd primary Kervaire elements,
(2) Brown-Gitler spectra and Mahowald's η_j-family,
(3) The Morava stabilizer group and its action on Lubin-Tate space,
(4) $K(2)$-local homotopy groups of certain Smith-Toda complexes.

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C O U R S E   D E S C R I P T I O N

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Spring 2019

MATH 595

L O C A L   C O H O M O L O G Y

Prof. S. P. Dutta

This course will be a study of Local Cohomology, introduced by Grothendieck, with various applications. The main topics will include: Cohen-Macaulay Rings and Modules, Injective Modules over noethierian rings, Gorenstein rings, local cohomology -- connection with dimension and depth, local duality theorem of Grothendieck, Cohomology of quasi-coherent and coherent sheaves, Serre's Theorem on coherent sheaves on projective spaces, classification of Line-bundles on P^n, Hartshorne - Lichtenbaum Theorem and Faltings Connectedness Theorem.

Prerequisite: Math 502

Recommended Text: 1. Local Cohomology by R. Hartshorne; 2. Local Cohomology by Brodmann and Sharp, Cambridge University Press.

Math 595 - Poisson Geometry - Spring 2020

This course is an introduction to Poisson geometry. Poisson geometry is the study of differentiable manifolds equipped with a Poisson bracket. Its roots lie in Classical Mechanics, but it became an independent field of study in the 70's and in the 80's, in parallel to its close cousin Symplectic geometry. If you have a basic knowledge of manifolds, vector fields and differential forms, you can get an idea of what Poisson geometry is by reading a brief introduction. In this course I will be following a book that I am writting with Marius Crainic and Ioan Marcut. You can check the table of contents. Students taking this course are assumed to know differential geometry at the level of Math 518 - Differentiable Manifolds.

Syllabus:

- **Basic Concepts.** Poisson brackets; Poisson bivectors; The Darboux-Weinstein Theorem.
- **The Symplectic Foliation.** Symplectic leaves and symplectic foliations; Poisson transversals; Symplectic realizations; Dirac geometry; Submanifolds in Poisson geometry.
- **Global Aspects.** Lie groupoids, integrability, symplectic realizations, averaging and linearization, Van Est map.
- **Symplectic Groupoids.** Complete symplectic realizations; Lie groupoids and Lie algebroids;Symplectic groupoids.
- **Special Topics.** To be chosen from: Moduli space of flat connections; A-symplectic structures; Conn's Linearization Theorem; Cluster algebras; Symplectic stacks; Symplectic foliations; Quantization deformation.

Textbooks:

I will provide some lecture notes as the course progresses, but the following two references should also be very helpful:

- A. Cannas da Silva and A. Weinstein, ***Geometric models for noncommutative algebras***, Berkeley Mathematics Lecture Notes, 10. American Mathematical Society, Providence, RI, 1999.
- J.-P. Dufour and N.T. Zung, ***Poisson Structures and Their Normal Forms***, Progress in Mathematics, Vol. 242, BirkhÃ¤user, Basel, 2005.

Grading Policy

- **Expository Paper:** Students will be encouraged to write (in LaTeX) and present a paper. This is not mandatory. Following the tradition of topics courses, there will be no homework and no written exams.

Mathematical Methods in Quantum Information Theory

Course: Math 595

Instructor: Marius Junge

Time: MWF 12

Course Description: Quantum information science may become the driving force for new technology in the near and distance future, and sounds like science fiction: 'spooky action at a distance', 'teleportation'. However, the math behind these physical phenomena is very real. In this course we will briefly review the foundations of quantum mechanics explaining measurements, observables, and states of a system. Then we will follow the guideline of classical information theory and discover channels, entropy and the difference between classical and quantum computation. Finally, we will cover a range of more advanced mathematical techniques in quantum information theory and matrix-valued functions.

Prerequisites: Math 540, and basic knowledge in complex analysis. A large part of quantum information theory deals with maps on finite dimensional Hilbert spaces, and draws from tools in linear algebra, analysis, and representation theory.

Books: (No need to buy them all)

i) Michael A. Nielsen & Isaac L. Chuang: *Quantum Computation and Quantum Information*, Cambridge University Press 2010.

ii) Mark Wilde: *From Classical to Quantum Shannon Theory*, Cambridge University Press, 2013.

iii) John Preskill: *Quantum Computation*, lectures notes from CalTech

iv) John Watrous: *The Theory of Quantum Information*, Cambridge University press 2018.

SPRING 2020
MATH 595
MODERN ALGEBRAIC GEOMETRY II

Time: Tuesdays and Thursdays 9:30–10:50
Room: TBA
Instructor: Sheldon Katz
Text: Algebraic Geometry, R. Hartshorne, Graduate Texts in Mathematics 52, Springer NY 1977
Prerequisites: Modern Algebraic Geometry I (Math 512) or equivalent, or permission of the instructor

This full-semester course will begin by covering most of the content of Chapter III of Hartshorne's text, especially derived functors and cohomology. The course will include extensive applications, following student and instructor interests. The text will be frequently supplemented with additional materials designed to enhance geometric intuition, and the treatment in class will often diverge from the text.

Students will submit a semester paper on a related topic of their choice subject to approval of the instructor.

Homework will not be assigned, although students are encouraged to work on the extensive collection of problems at the end of each section.

Math 595

Kay Kirkpatrick

Advanced Machine Learning

Prerequisites: Probability and Introductory ML

Description: Machine learning is a growing field at the intersection of probability, statistics, optimization, and computer science, which aims to develop algorithms for making predictions based on data. This course will cover foundational models and mathematics for machine learning, including statistical learning theory and neural networks, with a project component.

Text: TBD.