Course Description — Fall 2018

MATH 583

PARTIALLY ORDERED SETS AND MATROIDS

Section D1: MWF 9:00– 9:50 am, 145 Altgeld Hall.
Instructor: Professor József Balogh, 233B Illini Hall, jobal@math.uiuc.edu.
Office hours: TBA
Web page: http://www.math.uiuc.edu/~jobal/math583
TEXT: **D. B. West, The Art of Combinatorics, Volume III: Order and Optimization.** The current version will be at TIS Bookstore, 707 South Sixth Street, about $ 35. There also could be some handouts by instructor.

TOPICS: We discuss partially ordered sets and matroids, with applications to measuring preferences, sorting, searching, optimization, etc. Simple examples of *partially ordered sets* (posets) include a family of sets ordered by inclusion or a set of natural numbers ordered by divisibility. Additional topics might include applications of the Szemerédi Regularity Lemma and the hypergraph container method.

Poset structure: Dilworth's Theorem and generalizations, maximum antichains, symmetric chain decompositions, distributive lattices, comparability graphs. Dilworth's Theorem is a min/max relation: the maximum number of pairwise incomparable elements equals the minimum number of chains covering the elements. Lattices are posets having additional special algebraic structure.

Linear extensions: Preference relations, order dimension, geometric representations, correlational inequalities, sorting and searching. Arrow's Impossibility Theorem states that there is no way to aggregate the preferences of voters to satisfy four reasonable axioms. The dimension of a poset is the minimum number of linear criteria that determine it. Representations assign geometric sets to model the poset as an inclusion order. Correlational inequalities study events when the linear extensions are equally likely

Extremal problems: Extremal families of sets, intersecting families, on-line algorithms. What is the maximum size of a family of pairwise-intersecting subsets of an n-set? How to apply modern powerful tools, such as Szemerédi Regularity Lemma and the hypergraph container method.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 4 or 5 out of 6 problems. The problems require proofs related to or applying results from class. Roughly speaking, 80% of these points suffices for an A, 66% for a B. Discussions between students about problems can help understanding. Collaborations should be acknowledged, and submitted homework should be written (typed) individually. Electronic mail is a good way to ask questions about homework problems or other matters.

PREREQUISITES: Some familiarity with elementary combinatorics. Math 580 is ideal preparation, but any prior contact with partially ordered sets or graph theory may suffice. Similarly, 412 and 413 are sufficient prerequisites.