MATH 550 Comprehensive Exam ODEs

Do all four problems.

Problem # 1 Consider the ODE

$$\dot{x} = x \ln\left(\frac{1}{|x|}\right),$$

where the right hand-side is defined to be zero at $x = 0$. Consider the initial value problem $x(0) = 0$, which has a solution $x(t) = 0$.

- Does the uniqueness theorem apply here?
- If not, can you determine if this solution is unique?

Problem # 2 Let

$$\ddot{x} + a(t)x = 0$$

be a linear π-periodic ODE with $a = 1$ if $t \in (0, \pi/2)$ and $a = -1$ if $t \in (\pi/2, \pi)$.

- Find explicitly the period map.
- Determine the stability type of the map.

Problem # 3 Suppose A is a square matrix with eigenvalues having nonpositive real parts. Let $x(t)$ be a solution of the linear system $\dot{x} = Ax$.

- Give an example of A such that $x(t) \to \infty$ as $t \to \infty$.
- Show that if A is symmetric then $x(t)$ is bounded as $t \to \infty$.

Problem # 4 Consider the equation

$$\dot{x} = (\mu - x)(x^2 - 2x\mu + 1.)$$

Draw the complete bifurcation diagram.