525 Comp Exam.

Wednesday, May 16, 2016.

1. Given a map $f\colon X \to X$, define $C_f := X \times [0,1]/\sim$ where we identify $(x,1) \sim (f(x),0)$ for each $x \in X$.

   (a) Let $X = S^1 \subset \mathbb{C}$, and $f\colon X \to X$ by $f(z) = z$. Compute the homology of C_f.

   (b) Let $X = S^1 \subset \mathbb{C}$ and $f\colon X \to X$ by $f(z) = z^2$. Compute the homology of C_f.

2. Let (X, x_0) and (Y, y_0) be based spaces. Suppose both x_0 and y_0 admit neighborhoods for which there exist deformation retracts to $\{x_0\}$ and $\{y_0\}$ respectively. Let $V := X \coprod Y/\sim$ where we identify $x_0 \sim y_0$. Using only the Eilenberg-Steenrod axioms (Dimension, Sum, Exact sequence of a pair, Homotopy, Excision), compute H_*V in terms of H_*X and H_*Y.

3. Let $X \subseteq \mathbb{R}^2$ be the subset

$$X := S^1 \cup (\mathbb{R} \times \{0\}) \times (\{0\} \times [0, +\infty)),$$

   the union of the unit circle, x-axis, and the positive part of the y-axis.

   (a) Describe the fundamental group of X in terms of generators and relations.

   (b) Classify all 2-fold covering maps over X (not necessarily connected).