

525 Comp Exam
May 2019

1. (a) Compute $\pi_1(\mathbb{R}P^n, \infty)$ for all $n \geq 0$.
(b) Prove that any continuous map $f : \mathbb{R}P^3 \rightarrow S^1$ is homotopic to a constant map.
2. Prove that there is an isomorphism $H_*(X \times S^3) \approx H_*(X) \oplus H_{*-3}(X)$ for all spaces X and all $* \in \mathbb{Z}$.
3. Let $X = \mathbb{R}^2 \setminus \{(-1, 0), (0, 1), (1, 0)\}$. Give a classification of all 2-fold covering maps $p: Y \rightarrow X$ up to equivalence. Identify those p for which Y is a connected space. (Here we do not assume covering maps require connected spaces.)
4. Let X be the space obtained from an annulus $\{p \in \mathbb{R}^2 \mid 1 \leq |p| \leq 2\}$ by identifying each point (x, y) on the inner circle $C_1 = \{p \in \mathbb{R}^2 \mid |p| = 1\}$ with the point $(-2x, -2y)$ on the outer circle $C_2 = \{p \in \mathbb{R}^2 \mid |p| = 2\}$.
 - (a) Describe a CW-structure for X .
 - (b) Compute $\pi_1(X)$.
 - (c) Compute $H_*(X)$.