

525 Comp Exam

May, 2014

1. Let M be the Möbius band and ∂M its boundary. Compute the relative homology $H_*(M, \partial M)$.
2. Let $\mathbb{R}P^2$ be the two dimensional real projective space, which is obtained by attaching a 2-disc D by its boundary $S^1 \cong \partial D$ to a circle S^1 by the double covering map of S^1 to S^1 .
 - (a) Compute $\pi_1(\mathbb{R}P^2)$.
 - (b) Show that every continuous map from $\mathbb{R}P^2$ to $S^1 \times S^1$ is homotopy equivalent to a constant map.
3. Given an example of a connected space X with $\pi_1 X \neq H_1 X$. Be sure to fully justify your solution.
4. Let P be a plane and L a line not parallel to the plane. For $n \geq 3$ compute

$$H_*(\mathbb{R}^n - \{P \cup L\}).$$

5. Compute $H_*(X \times S^1)$ in terms of $H_*(X)$.