Topology Comprehensive Exam, May 2006

Problem 1: Show that if f is a continuous map from S^2 to S^2 such that $f(-x) = f(x)$ for all x, then $H_2(f) = 0$.

Problem 2: Consider a finite planar graph Γ as a one-dimensional cell complex within the plane R^2. Tell me everything you can about the homology of the quotient space R^2/Γ in terms of the topological properties of Γ. Take a guess as to how your answer changes if Γ is a non-planar graph in R^3 (no proof necessary for this last part...).

Problem 3: Show that every continuous map

$$f : RP^5 \to (S^1 \vee S^1) \times T^3$$

is homotopy equivalent to a constant map.

Problem 4: Let X be a closed 2-disc with two disjoint open 2-discs removed (see figure below). The three disjoint boundary circles are labeled a, b, and c as in the figure. Attach three disjoint closed 2-discs to X via a degree 3 map to a, a degree 5 map to b and a degree 15 map to c. Compute the homology of the resulting space.

Would your answer change if we permuted the labels a, b, and c in the figure below?