

Comprehensive Exam in Topology
University of Illinois, January 2010

1. (25 points) Let X be the union of the unit 3-sphere (centered at the origin) with the z -axis. Compute $\pi_1(X)$.
2. (25 points) Let X be the space obtained from a triangle PQR and its interior by identifying edges in the following way: identify \overrightarrow{PQ} with \overrightarrow{QR} , and identify \overrightarrow{PQ} with \overrightarrow{PR} . Compute H_*X .
3. (25 points) For each of the following either give an example or indicate why no example exists.
 - (a) A connected space X such that π_1X is a non-trivial finite group.
 - (b) A space X such that H_2X is a non-trivial finite group.
 - (c) A retraction from the 2-sphere to the circle on the equator.
 - (d) A self-map of the 2-sphere with no fixed points.
4. (25 points) Let $X = S^1 \vee S^1$ denote the one-point union of two circles. Classify all the 2-sheeted covering spaces over X , up to equivalence of covering spaces: draw a picture of each type of covering, and give a justification that these are the only ones.