Topology Comprehensive Exam in Topology

January 2006

1. Let $A \subset \mathbb{R}^n$ denote a finite set. Compute the homology of $\mathbb{R}^n$ relative to A, $H_*(\mathbb{R}^n, A)$, using any method you wish.

2. (a) Give an example of a compact space Y whose fundamental group $\pi_1 Y$ is uncountably infinite. (b) Prove that if X has a compact universal cover then $\pi_1 X$ is finite.

3. Let X be the quotient space obtained by the identification of an equator S^1 of S^2 with the diagonal $\Delta = \{(\theta, \theta) : \theta \in S^1\}$ of the torus $T^2 = S^1 \times S^1$. Compute the homology of X, $H_*(X)$, using any method you wish.

4. View the circle S^1 as the unit circle in $\mathbb{R}^2$. Show that any map $g : S^1 \to S^1$ such that $g(-x) = -g(x)$ must have odd degree.