

MATH 500 — MAY 2015

Four problems, 25 points each. Maximum 100 points.

1. (a) Give the definition of a nilpotent group in terms of its upper central series.
(b) Show that every p -group is nilpotent.
2. Show that A_5 does not contain a group of order 15.
3. (a) Show that any finite integral domain must be a field.
(b) Is the polynomial $p(x) = x^6 + x^3 + 1$ irreducible in $\mathbb{Z}[x]$?
4. Find the Galois group of the polynomial $p(x) = x^3 + 6x^2 - 9x + 3 \in \mathbb{Q}[x]$.