The Penrose Tiling of the plane is the most famous traditional "tiling" of the type which motivated this work. With just two tile types, a "kite" shape and a "dart" shape, and a corresponding subdivision rule, it provides fascinating tilings, ones in which the eye finds endless repetitions, but which in fact have no periodicities. Here we will discuss a new, even richer class of tilings, one in which conformal shapes replace the traditional euclidean shapes. The talk will intermingle the discrete world of circle packing with the continuous world of analytic maps and will convey—I hope—the pleasing blend of theory, computation, experimentation, and visualization that I have so enjoyed in this new topic. It will be a largely visual tour, so no background in tiling, circle packing, or analytic function theory is needed. (Much of this is joint work with Phil Bowers of Florida State University.)