The development of algebraic geometry has been motivated by enumerative geometric questions where one asks for the number of geometric figures of some type that satisfy a list of conditions. For example, the Gromov-Witten invariants of a flag manifold counts the number of curves that meet a list of Schubert varieties in general position. Many enumerative problems can be reduced to understanding the Schubert structure constants of flag manifolds. Standard conjectures about #P-functions indicate that these structure constants are best expressed as the number of objects in some combinatorially defined set. The classical Littlewood-Richardson rule for the structure constants of Grassmannians is an example of this, but it is not known if all Schubert structure constants can be (reasonably) expressed in this way. I will speak about recent results that express Schubert structure constants as the number of puzzles that can be created from a given list of puzzle pieces, as well as relations to the Gromov-Witten invariants of Grassmannians.