Pi Mu Epsilon at UIUC
Challenge Problem #4

March 1, 2013

The Stirling numbers of the second kind \( \{m \atop n\} \) count the number of ways to partition a set of \( m \) objects into \( n \) non-empty subsets, and satisfy the recurrence relation
\[
\{m \atop n\} = n \{m - 1 \atop n\} + \{m - 1 \atop n - 1\}
\]
with initial conditions
\[
\{m \atop n\} = \begin{cases} 1 & m = n \\ 0 & m \neq n, mn = 0 \end{cases}
\]

Prove that
\[
\left(\frac{n}{2}\right)^{m-n} \leq \{m \atop n\} \leq \left(\frac{m}{n}\right)^{m-n}
\]
where \( m \) and \( n \) are integers such that \( m \geq n \geq 0 \) and \( \binom{m}{n} \) is the usual notation for the binomial coefficient. Also show that we have the left equality if and only if \( m = n \), \( m = n + 1 \) or \( n = 0 \) and the right equality if and only if \( m = n \).