The models of fine art derivative

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Abstract

This article aims to use statistical models to give potential investors insights on fine art investment market since art pieces have become an increasingly popular portfolio diversification tools. In this article, two of the most popular pricing models, Hedonic Regression and Repeat Sales Regressions as well as their process of development is discussed. As the article aims to give intuition of fine art market to the art investors, it will discuss the advantages and disadvantages of fine art investment serving as a risk-aversion tool. Due to the nature of fine art market, we only obtain the information of Sotheby New York in 2000. By fitting Linear Regression, Hedonic Regression as well as Vine Copula to this data set, the accuracy is test to find the most applicable model. Lastly, as the art market is so inclusive and volatile, hedging tool is also introduced in the end of the article to help existing fine art investors.

Keywords: art index, pricing of fine art, fine art derivative

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1 Introduction

The art in its own nature is a fluctuating creation, its value changes in the society taste and ideological trend. In order to make art as a mature investment asset, there are two ways to counter the natural limits: globalization and research. As the art insurance premium would vary from factors that the artist is famous or not, the art's trading track, the material composition, etc. Art is also considered a prime resource that can repel the high inflation over the decades. The existing modern portfolio, which have run for at least 5 years, dictates that investors should have 5-7 percentage of their portfolios diversified into the art market. Most transaction of art are private sale, not many through public market. What's more, in short-term art is not a profit goods as stock or other financial instrument, which is why it is strategic to use market analysis to find the best time to sell. Not only we need professions in financing, but also art experts are required for a stable and selected portfolio.

As what market speculates, the gradually maturing trend in the investment of art leads to the concern of protection. There are many insurance companies took this opportunity and split into two categories: specialization in insuring art products, and combo policies for not only combined fine art's intrinsic value with its identities, but also treat arts derivative as a product as well. Collectible Insurance Services designs its policy to serve individual collectors and dealers, and it categorizes its product base on the type of art product. For individual collectors, their collection can be divided into six types: sports card and memorabilia; comics; guns, knives and accessories; art and fine art; toys; stamps. Collectible Insurance Services identifies the perils and exposure for each category and provide different quotes for each category. AIG also provides insurance policies based on different types of art product, including private collections, fine art, couture, jewelry, wine collection and art and antiques reservation expertise. The advantage of these companies is that for a peculiar art product they have a more precise pricing model and distributions of the severity and frequency for the risks of this type of art product. There are also companies which manage risks based on the owner who possess the art product rather than the difference of art product itself. For instance, Art Insurance Now provides insurance policy for art dealers and galleries, dealing with the risks during transportation worldwide and storages in multiple locations. It is also designed liability policy for auctioneers: coverage including consignments, estate sales, good in transit and special clauses for loss buy back or pairs and sets, etc. The advantage of these companies is that the personalized blanket coverage policies for different customers provide more convenient services for individuals.

Ultimately, there is something to remember, the asset of an art investment should value more than other expenses in the portfolio. For art is floating outside the primary and secondary market, thus it gives investors some intervals to free its capital at a right opportunity and use the money to save potential loss in other financial assets.

After introducing the fundamental structure of the general art insurance, our team takes a step forward to look into a specific area of art insurance—that is Regression Models. As quoted from Deloitte's research, the hedonic model could explain the reasonable pricing for an art up to 88 percentage. This fact reflects that the priceless is not necessarily im-priceable. To achieve that, we format a general timeline for the art price index. From Goetzmann, who used simple models to compound the value of art by cumulating interest rate, to Pesando and Shum, who develop a higher price art dominates model in art auctions.

Art investment has become a hot topic in alternative investment since the last economic crush in 2008. Its unique characteristics, different from other traditional financial tools, has given investors hope that the fine art investment may become a way to hedge their investments in the next round of financial downturn. However, some of the unique properties of art such as heterogeneity have also stopped many potential investors. Therefore, price index is demanded to enable simple and easy comparison between traditional financial instruments and fine arts. In this article, we are going to analyze two of the most popular calculations behind the art price indices Hedonic Regression and Repeat sales regressions as well as its development over time. So later on, the report will demonstrate the most practicable models and the crucial parameters for valuing single art products that our team has figured out. Moreover, we choose to use the data from the sotheby in New York to fit in the model and calculate the art index during 2011-2016.

In general, the rate of return of fine art is modest in comparison with other financial tools. There are some discrepancies between conclusions drawn from different researchers. However, the reasons are pretty clear too. Mainly due to the difference in perspectives, market and time differences that different researchers took. In addition, the claim on the arts ability to diversify the portfolio was also proven as false due to the long-term correlation between art and economic market.

2 Repeat sales regressions

Repeat sales regressions are used in numerous articles that estimate price indexes from art auctions including Pesando [1993], Goetzmann [1993], Mei and Moses [2002] and Pesando and Shum [2008]. Ashenfelter and Graddy [2003, 2006] provide a survey. Most of the articles on art auctions do not provide the results from the second stage regressions and Pesando [1993] simply uses ordinary least squares (OLS). As art is an infrequently traded asset with many of the same properties as musical instruments and housing, it is very likely that many of these studies could benefit from the non-parametric specification in the second stage of the repeat sales regressions.

2.1 Goetzmann [1993]

First, we assume that $r_{i,t}$ which is the continuously compounded return for a certain art asset i in period t, could represented by μ_t and an error term.

$$r_{i,t} = \mu_t + \epsilon_{i,t},$$

where μ_t is the continuously compounded return of a price index of art, may be thought of as the average, return in period t of paintings in the portfolio and $\epsilon_{i,t}$ is the error term, We will use sales data about individual paintings to estimate the index μ over some interval,t = 1. T; and μ is a T-dimensional vector whose individual elements areμ_t; r is N-dimensional vector of logged price relatives for N repeated-sales observations; $P_{i,b}$ is the purchase price; $P_{i,s}$ is the sale price; b_i is date of purchase; s_i is date of sale,

So, the logged price relative for asset i, held between its purchase date b_i, and its sales Date s_i, may be expressed as

$$r_t = ln(P_{i,s}/P_{i,b}) = \sum_{t=b_i+1}^{s_i} r_{i,t} = \sum_{t=b_i+1}^{s_i} \mu_t + \sum_{t=b_i+1}^{s_i} \epsilon_{i,t}.$$

Because of the generalized least-squares, the maximum-likelihood estimate ofμ is the form:

$$\hat{\mu} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}r,$$

where X is an $N * T$ matrix, which has a row of dummy (fake) variables for each asset in the sample and a column for each holding interval; and Ω is a weighting matrix, whose

weights could be set as the times between sales.

2.2 Mei and Moses [2002]

In Mei and Moses's paper, they use the same model that Goetzmann built. But they talked more about how to deal with the biases about the phony negative autocorrelation. In the estimated series, RSR also have some biases in the estimated series. And the most important bias is a phony negative autocorrelation when estimate the return . Goetzmann (1992) proposed a two-stage Bayesian regression to mitigate this negative autocorrelation. The Bayesian formulation has an additional restriction which is the return series is distributed normally and is identically and independently distributed.
And the form of the Bayesian estimator is:

$$\mu_{bayes} = [(X'\Omega^{-1}X) + k(I - \frac{1}{T}J)]^{-1}X'\Omega^{-1}r,$$

where $k = \frac{\sigma^2}{\sigma_\mu^2}$.

2.3 Pesando and Shum [2008]

Same model as Mei and Moses'[2002] and they used data from to test the model and get some result about the relationship between return and the price of fine arts.

The most high-priced prints, around top 5 percentage by price, have outperformed in the whole market. However, there is no evidence to proof that, if all other things are equal, it is always better to purchase a more expensive work of art than a cheaper one. The price of a fine art is not determined by a future stream of payments, this is the main difference between the traditional financial asset and the fine art asset. What's more, the price of fine art may fluctuate because of essentially random changes in tastes. Pesando and Shum [2008], for artists who are acknowledged masters of the 20th century, there is evidence show that : the prints of Picasso significantly outperform those of Chagall and (especially) of Miro during 1977 to 2004.

3 Hedonic Regression for Art Price Indices

3.1 Pricing Artwork Using Hedonic Regression

Recently artwork is becoming a more popular investment and hedging tool due to the downfall of the financial market. In order to measure the financial performance of the art pieces, different models are employed to find the most accurate price estimation, for example, Repeat-Sales Regression, Hedonic Regression, Naive Art Price Indices, etc.

Hedonic Regression is one of the most commonly used models in the market. Its ability to take in all sales records is the first reason why Hedonic Regression is so popular among scholars. Being able to take all sales records into consideration first shows a great presentation of the whole art market, an objective measurement of the performance of certain artwork. Since Hedonic Regression takes the characteristics of the artworks into consideration while building models, this effectively avoids the problem of not considering the unique characteristics of art pieces while measuring the changes in its prices. As time and owners of those art works change, some qualities of artworks also change depending on how the owners are taking care of these works as well as the characteristics of the artwork itself. Applying Hedonic Regression and taking all properties into consideration separately allows researchers to monitor these changers closely as well as measuring the art price indices for a certain submarket. Therefore, making more accurate prediction with small variance. Lastly, by focusing on the individual properties of the art pieces and including the information of buyers of those artworks, scholars can capture the willingness of potential investors to pay for certain fine arts.

However, Hedonic Regression is still not the perfect model to use in measuring the performance of the artworks. The main problem of Hedonic Regression is that the model is strongly influenced by the characteristics of the artworks chosen. Therefore, wide experience and expert knowledge is required to apply the Hedonic Regression, but selective bias still cannot be fully eliminated. In some extreme cases, when the information on the characteristics of the artworks is limited, Hedonic Regression then is not a good instrument to be applied.

Thanks to the fast connection created by the Internet, some of the limitations of Hedonic Regression have been overcome. For example, the information on the artworks is now much commonly available and understandings towards fine arts are better developed. Therefore,

the situation is becoming more applicable to the application of Hedonic Regression. In addition, a more objective system in taking characteristic variables is starting to develop among scholars.

3.2 Basic Hedonic Regression

The basic form of Hedonic Regression is developed by taking characteristics of art works as well as the time of sales as dummy variables and fitting a regression using the natural log of the price of fine arts. It is usually represented by the equation below:

$$\ln\left(P_{kt}\right) = \alpha_0 + \sum_{j=1}^{x} \beta_j X_{nkt} + \sum_{t=1}^{t} \lambda_t C_t + \epsilon_{kt} \quad with\ \epsilon\ N\left(0, \sum_{k} \Theta I_t\right),$$

where $P_{kt} = f(x_{1,kt}, x_{2,kt}, x_{x,kt})$ is a vector of the sales prices, α_0 represents the regression intercept, β_j is the coefficient value for characteristic x, X_{nkt} is the characteristic value of artwork n, λ_t is the coefficient value for time-dummy t, C_t reflects the time period value that can be 0 or 1, and ϵ_{kt} is the disturbance term.

Some common characteristics included in Hedonic Regression are sales dates, medium, auction house, surface of the art product, existence of signature, estimation price of the artwork, the living status of the artist as well as his/her reputation. In Roman Kraeussl and Robin Loghers article Emerging art markets, they defines five difference factors as the dummy variables as following.

3.2.1 Auction House

Due to the difference in location and reputation of different auction houses, the price of the artwork also various. The auction house variable is treated as a dummy variable, such that 1 means specific painting has sold through a particular auction house.

3.2.2 Medium

Different media and material used is proved to have significance in affecting the final price of the artwork. For example, it is commonly understood that sculpture worth higher value than painting. The variables again are specified as dummy variables, such that 1 indicates that a painting has a certain combination of medium and material, and other stands for all other materials, excluding those specified as regression variables and the reference variable, oil on canvas.

3.2.3 Surface

The surface means the size of the artwork. This size variable usually denotes the height, width and length of the artwork. The surface usually is positively correlated with the price of the artwork. However, there is a diminishing effect on the surface variable too, as huge artworks cannot be easily stored and present in a single room.

3.2.4 Estimate price

Estimate price is also a dummy variable, indicating if there is estimation on the price of the artwork. The artworks without estimation price usually sold at higher price than the artworks with estimation price, since investors tend to be more conservative towards the price of artworks with estimation.

3.2.5 Signature

A painting with signature of artist provides a sense of authentication towards the investors. Therefore, they are usually sold at a higher price than the artworks produced by same artist without signature. Signature is again a dummy variable with 1 presenting lack of any sign of authenticity.

4 Hybrid Approach

In order to counter the limitations in both Hedonic Regression and Repeat-Sales Regression, Locatelli Biey and Zanol borrowed the concept of Case and Quigley in 2005, which was initially implemented to target the real estate market. The Hybrid Approach allows researchers to capture the movement in the price of artworks within the Repeat-Sales Regression as well as the correlations between the characteristics of art pieces like Hedonic Regression do. Therefore, it is able to include all fine arts in current market for art pricing and improve the volatility in price estimation as Hedonic Regression do and finally overcomes these limitations of Repeat-Sales Regression. However, as Repeat-Sales Regression does, the Hybrid Approach is subject to sample selection bias too.

Locatelli Biey and Zanol [2005] first priced the artworks using Hedonic Regression with Hybrid Approach as the equation listed below:

$$ln(P_{it}) = X_i\alpha + B_{it}\beta + D_{it}\delta + \varepsilon_{it},$$

where P_{it} is the selling price of the i-th printing at time t; X_i is the set of relevant time-invariant chacteristics of i-th printing; B_{it} is the set of relevant time-varying characteristics; D_{it} is a set of dummy variables with a value of one if the selling of i-th painting occurs at time s and zero otherwise, ε_{it} is the disturbance term, and α, β, δ are the shadow prices of the ith painting characteristics.

Then they restrict the sample data to artworks that have been sold for more than once, so that Repeat-Sales Regression can be applied to the data set as following:

$$P_{i(t+s)} - P_{it} = A_{i\tau}\beta + T_{i\tau}\delta + \nu_{i\tau},$$

where

$$A_{i\tau} = B_{i(t+s)-}B_{it}, \nu_{i\tau} = \varepsilon_{i(t+s)} - \varepsilon_{it},$$

and

$$T_{i\tau} = \begin{cases} 1 \quad if \quad T = t+s, \\ -1 \quad if \quad T = t \\ 0 \quad otherwise. \end{cases},$$

However, there is not sufficient information can be drawn from the current market to estimate the parameter β. Hence, the cross equation equality constraints are imposed on β and δ in order to obtain smaller standard deviation observations.

$$\begin{bmatrix} P_{is} \\ \Delta P_{iT} \end{bmatrix} = \begin{bmatrix} X & B & D \\ 0 & A & T \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \delta \end{bmatrix} + \begin{bmatrix} \epsilon \\ \upsilon \end{bmatrix},$$

In Locatelli Biey and Zanols article [2005], a Hybrid Model Approach, four characteristics of Picasso printing are taking into considerations, which are Surface, Number of prints, Signature, Color, Auction House, Media and Period.

4.1 Surface

Similar to Roman Kraeussl and Robin Logher [2012], the variable surface also represents the size of printing. Number of prints represents any identical copies present in the market.

4.2 Signature

Similar to Roman Kraeussl and Robin Logher, it represents that if there is Signature on the painting. Signature=1 means the painting is signed by Picasso, Signature=0 means it is not.

4.3 Color

Color is a dummy variable represents the colors of the printing. If the printing is printed with more than one color, color=1; otherwise, color=0.

4.4 Auction House

Similar to Roman Kraeussl and Robin Logher, the Auction House and location of the auction of the painting being sold are also taking into consideration.

4.5 Media

The understanding towards Media between Roman Kraeussl, Robin Logher and Locatelli Biey, Zanol are slightly different. Locatelli Biey and Zanol believe that Media reflects the techniques used by the artist on his painting. These techniques influence the price of the painting.

4.6 Period

Lastly, the time dummy variable is Period. It is included for each semester from 1988 to 1995.

4.7 Conclusion of Hybrid Approach

In their article, Locatelli Biey and Zanol concluded that even though the Hybrid Approach is a new strategy to improve the price estimation, it is not a great choice for reality application due to its hardness in identifying the dummy time variables. Although according to this data set, the results show the ability of Hybrid Approach in reducing volatility in estimation. However, the low quality in the data set restricts scholars to further establish confident conclusion on the estimation of general art market. From Locatelli Biey and Zanols conclusion, it is clear that Hybrid Approach is not enough developed to be ready to be applied to the market and still highly relied on the quality of the data set collected.

5 Price Index Calculation

All the different regression models help to estimate the price of certain art piece given the conditions of the artwork. In order to measure the performance of the potential investable art piece in the market, the price index is calculated through $\widehat{\beta_j}$ and $\hat{\lambda}_t$ estimation. Following we are going to study some of the price index calculation models analyzed by Witkowska Dorota and Kompa Krzysztofs article Constructing Hedonic Art Price Indexes for the Polish

Painting Market using Direct and Indirect Approaches as well as Roman Kraeussl and Robin Loghers article Emerging art markets.

5.1 Direct Approach

One of the easiest ways in price indices calculation introduced by Witkowska Dorota and Kompa Krzysztof [2014] in their article is to treat time dummy variable as a subgroup of the different art characteristics, since it is affecting the price of artwork as all properties do. This method is applied by Ginsburgh [2005] as well as Renneboog and Spaenjers [2013]. The formula of the Hedonic Total Index (HTI) in the period t is listed as following:

$$HIT_t = e^{\hat{\lambda}_t}.$$

In Ginsburghs article [2006], he also introduced another hedonic regression without intercept term. In this case, a similar calculation is applied. And

$$HI_t = \frac{e^{\hat{\lambda}_t}}{e^{\hat{\lambda}_{t-1}}}$$

for $t = 1, 2, , T$ and $HI_0 = 1$. This model of calculation is widely applied for Citadel Art Price Index and some other scholars.

5.2 Indirect Approach

Different from the direct approach, the indirect approach consider the time dummy variable separately from the other property variables. Hence the hedonic quality adjustment (HQA) is calculated based on the direct approach model with consideration of $\hat{\lambda}_t$ estimation. It indicates any changes in the properties of the artworks and is defined as follows:

$$HQA_t = exp[\sum_{j=1}^{k} \beta_t (\sum_{i=1}^{n} \frac{X_{ij,t}}{n} - \sum_{i=1}^{m} \frac{X_{ij,t}}{m})],$$

where $X_{ij,t}$ represents the j-th characteristic of the i-th artwork in time t, n and m are number of lots sold in time t and t-1 respectively.

Then Hedonic Index (HI) is calculated as following:

$$HI_t = \frac{NI_t}{HQA_t} = \frac{\prod_{i=1}^{n} (P_{i,t})^{1/n} / \prod_{i=1}^{n} (P_{i,t-1})^{1/m}}{HQA_t},$$

where NI_t is the nave price index at time t, $P_{i,t}$ is the price of the i-th artwork at time t.

5.3 Two-Step Hedonic Regression

Reputation of the artist is highly positively correlated to the price of the artwork. Artists reputation presents the recognition of the market towards artists work and taste. This variable allows the investors to distinguish well-achieved artists from ordinary ones. However, Reputation can be ambiguous to be measured. In Emerging Art Markets, Roman Kraeussl and Robin Logher applied a comparison technique to quantify this variable and applied it to two-step hedonic regression.

In two-step hedonic regression, the standard hedonic regression is calculated using the previous equation without reputation variables. In second step, we then can calculate the index using the formula below:

$$Index_y = \frac{\prod_{i=1}^{n} (P_{i,y})^{1/n}}{e^{\sum_{j=1}^{x} \beta_j (\sum_{i=1}^{n} \frac{X_{ij,y}}{n} - \sum_{i=1}^{m} \frac{X_{ij,r}}{m})}} / \frac{\prod_{i=1}^{m} (P_{i,y})^{1/m}}{e^{\sum_{j=1}^{x} \beta_j (\sum_{i=1}^{n} \frac{X_{ij,y}}{n} - \sum_{i=1}^{m} \frac{X_{ij,r}}{m})}},$$

where P_y is the sale price of a price of a particular artist y, P_r indicates the sales price of the reference artist r, n is the number of a paintings by a particular artist y, m is the number of painting by the reference artist r, β_j refers to the regression coefficient of a particular quality characteristic j, $X_{ij,y}$ is the particular paintings quality characteristic value for the artist y, and $X_{ij,r}$ indicates the particular paintings quality characteristic value for the reference artist r.

6 Advantage and Disadvantage of Fine Art Investment

There are many reasons behind fine art investment. However, is fine art a good investment tool to achieve the financial goal of all investors? Just like there are two sides of a coin, there are definitely advantages and disadvantages depending the perspectives taken by the investors. In the following paragraph, the articles published by J.P Morgan and many other papers are analyzed to exam the pros and cons of fine art investment.

6.1 Advantage

According to The European Fine Art Fair (TEFAF), the size of the global art market was roughly US 56 billion in 2012, which was 6 times as big as it was 20 years ago. The one reason behind the significant growth other than aesthetic satisfaction is the historical returns of art investment. The volatility of art pieces was less than equities and commodities according the article The Art of Investing in Art published by JP Morgan. The high long-term return and less volatile characteristics encourages the investors to invest in fine art pieces.

In addition, fine arts have little or even negative correlation with other financial tools such as equity and bond as well as its ability to hedge against inflation. In order to minimize exposure to unsystematic risk, investors will try to diversify their portfolios as much as possible by choosing uncorrelated stocks and equity. Art assets offer same characteristics too. According to figure 1 below: the chart Low or Negative Correlations of Art to other Asset Classes (1988 to 2012) published by JP Morgan, the correlations between Mei Moses World All Art, which calculates the return of reselling painting over last 50 years annually, and all other financial index are very small or even negative.

Investors also willing to invest in fine arts under inflation condition as it is more insensitive to the deflation and able to yield the best in comparison with other financial engines.

6.2 Disadvantages

Although Fine Art sounds like a perfect tool for risk diversification and yielding long-term return, it embedded with a lot of potential risks too. The murky nature of the art market limits the entrance of armatures. The conflict in interest of art market players can highly impact on the price of the art piece as well as the transition cost, not even mentioning the insurance and administrative costs.

However, the most important factor that making fine art not ideal financial tool is it does not follow the traditional analysis of volatility returns. The value of fine art volatiles when different pricing approaches is used and the change in the confidence of art investors in the market. Under the paper pricing the art piece using Hedonic Regression, the prices of fine arts is highly correlated with the traditional financial market, making it an unideal diversification tool.

7 Alpha Hedging

As proven by all the earlier sections, fine art investments are quite another thing in comparison with the traditional stock investment. Instead of changing with the volatile interest rate, fine arts are more vulnerable to the confidence of art investors towards either the artist or the characteristics of the art itself in the art market, i.e. strong future confidence leads to high rate of return eventually. Performing fine art hedging is mostly hedging against the volatility of market confidence, which is also called as alpha hedging.

In order to conduct alpha hedging, the general market confidence was needed. The Art-Tactic Art Market Confidence Index (AAMCI), denoted by λ_A, was invested to give investors a better tool to understand how popular is a certain art piece/ a particular artist is in the current art market. Over 100 individuals, with occupations in every aspect of fine art market, were surveyed to exam the popularity of the art pieces. For the seek of clarity, in this case, a classification of art works or an artist is measured, but not a single piece of fine art.

According to alpha hedging, the ${}^{h}\alpha$ is the return volatility of the artwork in proportion of the market confidence. There is no way to determine the future volatility of fine arts. However, we can use historical volatility calculated from the past selling prices instead.

As there is no dividend payment for fine arts, the rate of return on fine art investment can be defined as following:

$$u_i = ln\left(\frac{S_i}{S_{i-1}}\right), for\ i = 1, 2, \ldots, n,$$

where u_i is the rate of return and S_i is the price of fine art at time i.

The sample standard deviation of rate of return, s, on $N+1$ observations is:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} u_i^2 - \frac{1}{n\,(n-1)}(\sum_{i=1}^{n} u_i)}.$$

The sample historical volatility is:

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}},$$

where τ is the interval year-length.

With all these information available, the alpha hedging can be calculated as

$${}^{h}\hat{\alpha}_y = \frac{\hat{\sigma}}{\lambda_{Ay}},$$

where y is number of year, λ_{Ay} is the annual AAMCI of τ_y.

If we further assume that the stock price follows Geometric Brownian Motion,

$$ln(\frac{S_T}{S_0})\ N((\mu - 0.5\sigma^2)T, \sigma^2 T),$$

where $\sigma$is the annual volatility of art return and μ is the expected value of return.

Given the confidence interval (p), we are able to calculate the possible interval of price of art piece in given time interval,

$$e^{lnS_0+(\mu-0.5\sigma^2)T-N^{-1}(p))\sigma T^{0.5}} < S_T < e^{lnS_0+(\mu-0.5\sigma^2)T+N^{-1}(p))\sigma T^{0.5}}.$$

Under alpha hedging, the price range then becomes

$$e^{lnS_0+(\mu-0.5\sigma^2)T-N^{-1}(p))\alpha} < S_T < e^{lnS_0+(\mu-0.5\sigma^2)T+N^{-1}(p))\alpha}.$$

After knowing the possible range of the price of fine art, we then can hedge the risk by purchasing or selling derivatives accordingly using the Black-Scholes formula with previous assumptions remain as true.

For example, the price of a call option will be calculated as following:

$$C(S_T, T) = S_T * N(d_1) - K * exp[-rT] * N(d_2),$$

where K is the strike price at time T,

$$d_1 = \frac{ln(\frac{S_T}{K}) + (r + (\sigma^2)/2)(T)}{\alpha},$$

and

$$d_2 = \frac{ln(\frac{S_T}{K}) - (r + (\sigma^2)/2)(T)}{\alpha}.$$

8 Fit model

```
#Installing Package#
#install.packages("splitstackshape")
library(splitstackshape)
library(caret)
library(stringr)
library(FactoMineR)
library("factoextra")
library(copula)
library(VineCopula)
```

```
#Data Cleaning#
art = cSplit(art,"Size", sep = ",")
art$Sold.For = as.numeric(gsub('[,USD Premium]', '', art$Sold.For))
art=art[!is.na(art$Sold.For),]
art$Misc. = as.numeric(art$Misc.)
art$year = str_extract(art$Sale.of, "[0-9][0-9][0-9][0-9]")
art$Signature <- vector(length = length(art$Misc.))
for (i in 1:length(art$Misc.)) {
art$Signature[i] <- if (art$Misc.[i] == 1) 0 else 1
art$Estimation[i] <- if (art$Estimate == '') 0 else 1
}
```

```
art$Estimation=as.factor(art$Estimation)
art$Area = art$Size_1*art$Size_2
art$Area[is.na(art$Area)] = 0
art$Misc. <- NULL
art$Estimate <- NULL
art$Size_1 <- NULL
art$Size_2 <- NULL
art$log_price=log(art$Sold.For)
art_sig_num=pobs(art[,c(2,7,9,10)])
```

In this case, the variables: name of the artists X and Title are removed for redudancy. Since all 2100 observations are sold at Sothebys New York, the variable Sale.of is also removed. This is also applied to the variable year since only art work been sold at year 2000 is included for model construction. Lastly, since all the art works have an estimation price, it is not signficant variable. Therefore variable Estimation is also removed.

```
Art_reduction = lm(Sold.For~ Medium+Signature+Area, data=art)
summary(Art_reduction)
```

R output:

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '␣' 1
##
## Residual standard error: 337500 on 1480 degrees of freedom
## Multiple R-squared: 0.06255, Adjusted R-squared: -0.04007
## F-statistic: 0.6095 on 162 and 1480 DF, p-value: 1
```

From the summary above, we can see that Medium ‘Oil on Panel‘and variable ‘Signature‘ are signficant variables with respect to the dependent variable Sold.for under the significance level of 0.05. However, the model still is not able to explain the data set well since the ‘Multiple R-squared is only 0.07354 and the ‘Adjusted R-Sqaured‘ is -0.05397.

```
par(mfrow=c(2,2))
plot(Art_reduction)
```

R output:

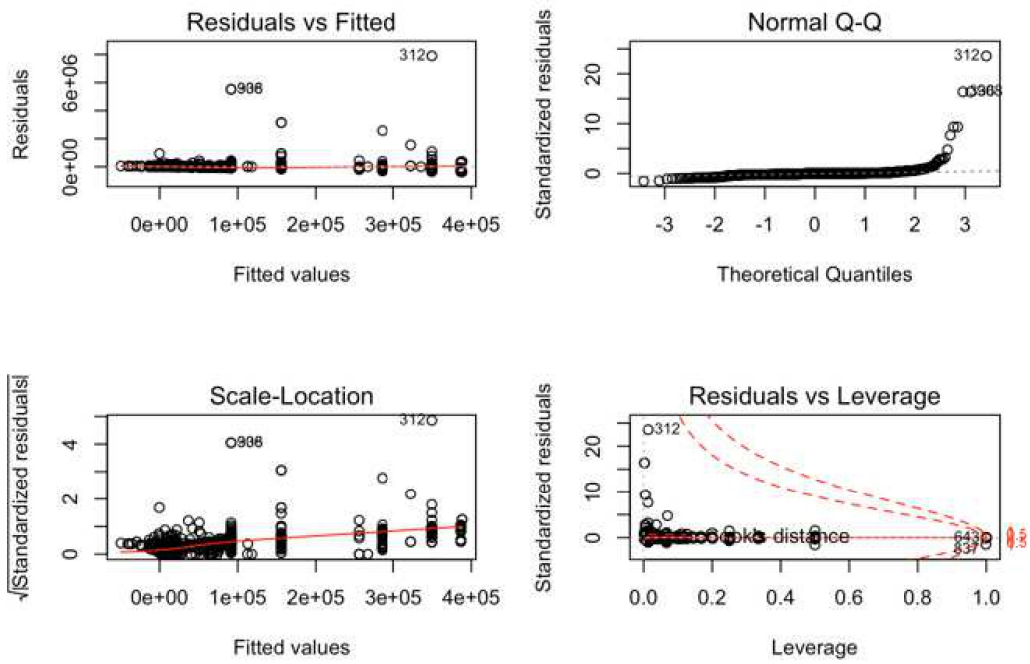

Figure 1

From the plots above, we can see the there are still a lot of leverage points in the data set. Hence we can conclude that linear regression model is not a good fit and should move on to find a better model for the price prediction.

```
#Fitting lognormal linear regression model#
Art_log = lm(log_price~ Medium+Signature, data=art)
summary(Art_log)
```

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '␣' 1
##
## Residual standard error: 1.061 on 1481 degrees of freedom
## Multiple R-squared: 0.491, Adjusted R-squared: 0.4356
## F-statistic: 8.872 on 161 and 1481 DF, p-value: < 2.2e-16
```

For the linear regression model with log(Sold.for) is working much better, there are 47 variables significant to log(Sold.for) at significant level of 0.05. In addition, the Multiple R-squared is now 0.5066 and Adjusted R-squared is 0.4397, which means the model is now able to explain 50.66 percentage of the data set.

```
par(mfrow=c(2,2))
plot(Art_log)
```

R output:

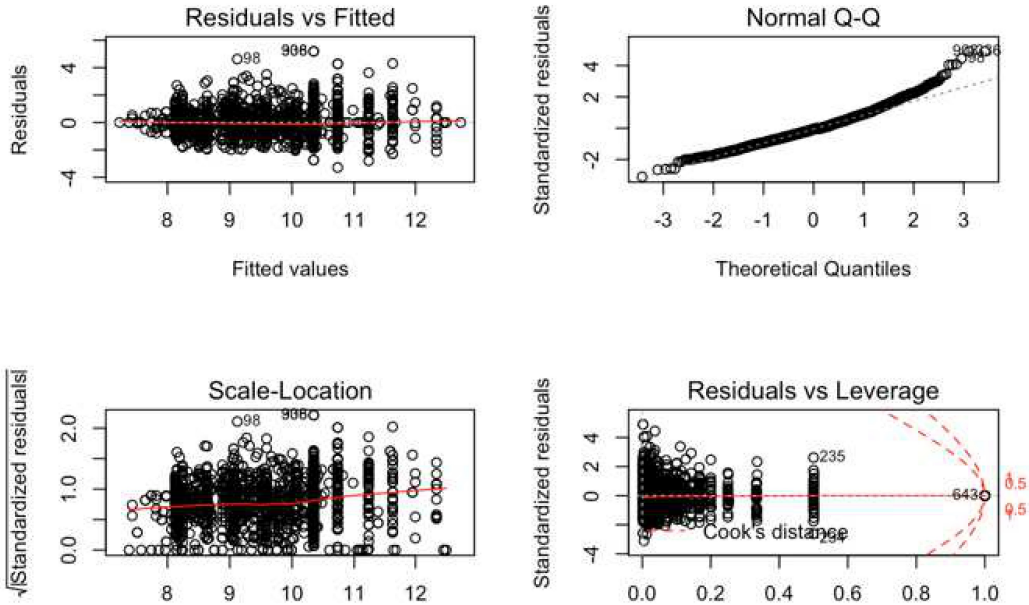

Figure 2

From the digrams above, the model is much more improved as there are less leverage points presented. In the residuals plot, the residuals are consistent across all fitted values.

```
#Vine Copulas with Gaussian copula#
rvm= RVineStructureSelect(art_sig_num, type = "RVine",familyset = 1)
goodftest = RVineGofTest(art_sig_num, rvm)
goodftest$p.value
```

```
[1] 0.725
```

The 'null hypothesis' is $H_0 : H_\theta + C_\theta = 0$ and the 'alternative hypothesis' is $H_0 : H_\theta + C_\theta \neq 0$,

where H_θ is the expected Hessian matrix and H_θ is the expected outer product of the score function.

Since the ‘p-value‘ is larger than $\alpha = 0.05$, we fail to reject the null hypothesis at significant level of 0.05. Hence we can conclude it is a suitable model.

```
RVineTreePlot(rvm, tree=1, 2)
```

R output:

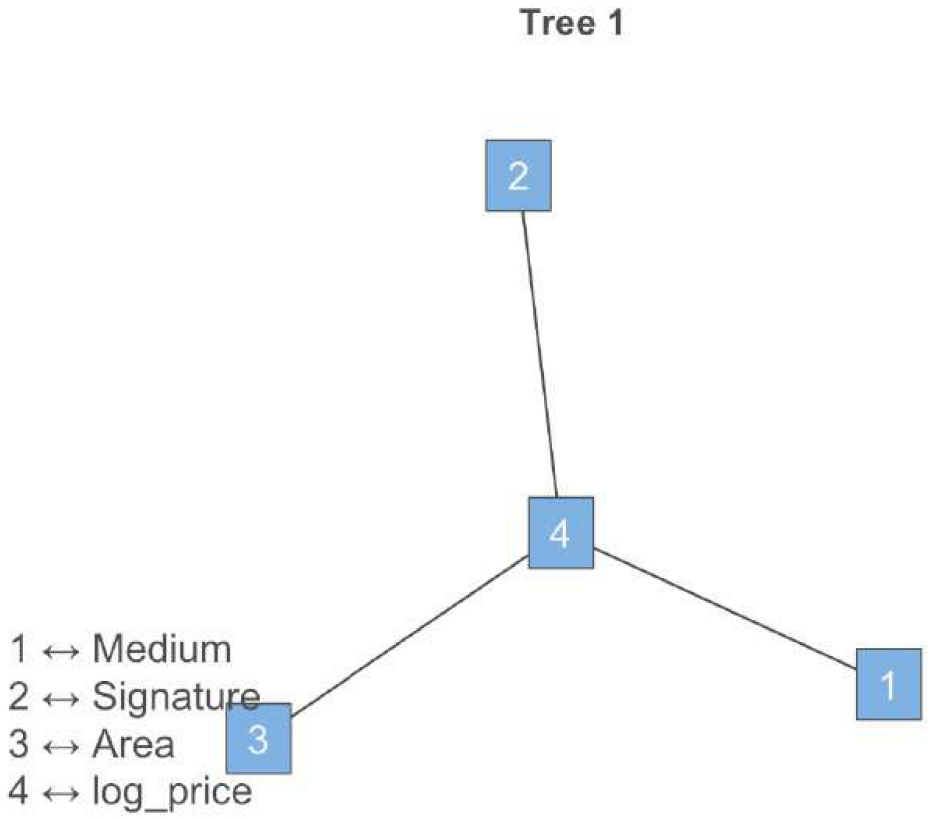

Figure 3

```
RVineTreePlot(rvm, tree=2, 2)
```

R output:

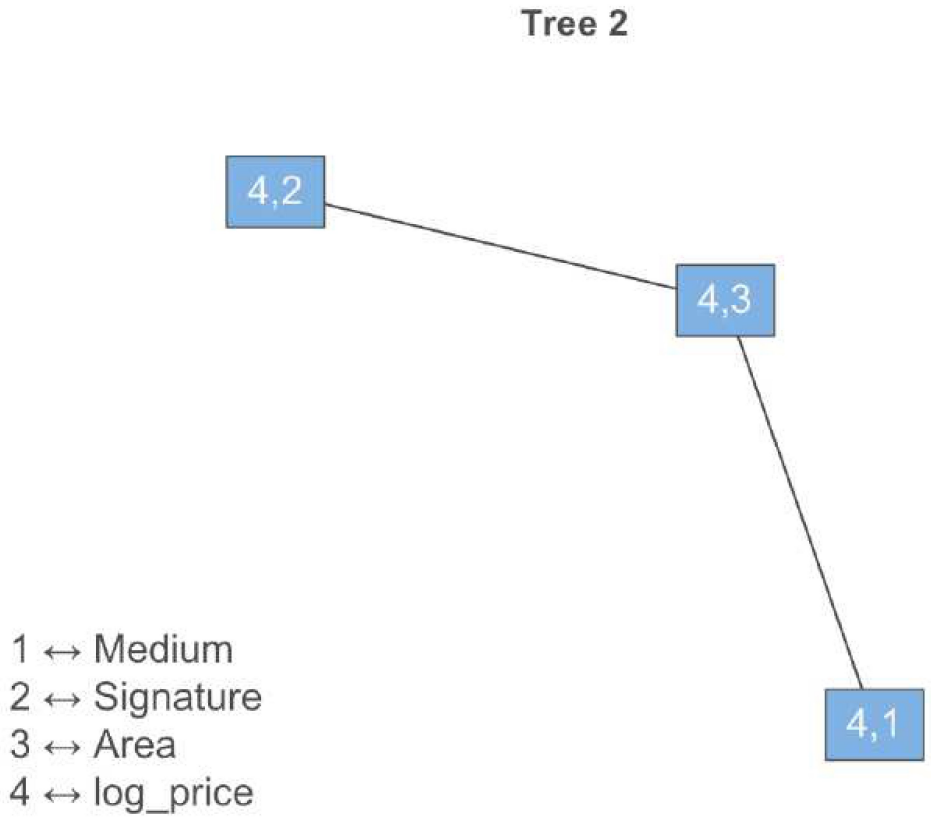

Figure 4

A Appendix

```
#Reading raw data#
art_1 = read.csv("file1.csv")
art_2 = read.csv("file2.csv")
art_3 = read.csv("file3.csv")
art_4 = read.csv("file4.csv")
art_5 = read.csv("file5.csv")
art_6 = read.csv("file6.csv")
art_7 = read.csv("file7.csv")
art_8 = read.csv("file8.csv")
art_9 = read.csv("file9.csv")
art_10 = read.csv("file10.csv")
art_11 = read.csv("file11.csv")
art_12 = read.csv("file12.csv")
art_13 = read.csv("file13.csv")
art_14 = read.csv("file14.csv")
art_15 = read.csv("file15.csv")
art_16 = read.csv("file16.csv")
art_17 = read.csv("file17.csv")
art_18 = read.csv("file18.csv")
art_19 = read.csv("file19.csv")
art_20 = read.csv("file20.csv")
art_21 = read.csv("file21.csv")
art = rbind(art_1, art_2, art_3,art_4, art_5, art_6, art_7,
art_8, art_9, art_10, art_11, art_12, art_13, art_14, art_15,
art_16, art_17, art_18, art_19, art_20, art_21)
#Installing Package#
#install.packages("splitstackshape")
library(splitstackshape)
library(caret)
library(stringr)
library(FactoMineR)
library("factoextra")
library(copula)
```

```
library(VineCopula)
#Data Cleaning#
art = cSplit(art,"Size", sep = ",")
art$Sold.For = as.numeric(gsub('[,USD_Premium]', '', art$Sold.For))
art=art[!is.na(art$Sold.For),]
art$Misc. = as.numeric(art$Misc.)
art$year = str_extract(art$Sale.of, "[0-9][0-9][0-9][0-9]")
art$Signature <- vector(length = length(art$Misc.))
for (i in 1:length(art$Misc.)) {
  art$Signature[i] <- if (art$Misc.[i] == 1) 0 else 1
  art$Estimation[i] <- if (art$Estimate == '') 0 else 1
}
art$Estimation=as.factor(art$Estimation)
art$Area = art$Size_1*art$Size_2

art$Area[is.na(art$Area)] = 0
art$Misc. <- NULL
art$Estimate <- NULL
art$Size_1 <- NULL
art$Size_2 <- NULL
art$log_price=log(art$Sold.For)
art_sig_num=pobs(art[,c(2,7,9,10)])
Art_reduction = lm(Sold.For~ Medium+Signature+Area, data=art)
summary(Art_reduction)
par(mfrow=c(2,2))
#Fitting lognormal linear regression model#
Art_log = lm(log_price~ Medium+Signature, data=art)
summary(Art_log)
par(mfrow=c(2,2))
plot(Art_log)
#Vine Copulas with Gaussian copula#
rvm= RVineStructureSelect(art_sig_num, type = "RVine",familyset = 1)
goodftest = RVineGofTest(art_sig_num, rvm)
goodftest$p.value
RVineTreePlot(rvm, tree=1, 2)
```

```
RVineTreePlot(rvm, tree=2, 2)
```

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