UNIVERSITY OF ILLINOIS

DEPARTEMENT OF MATHEMATICS

ACTUARIAL SCIENCE

Solvency II for an insurance company in Europe

Prepared by

Changyu Yin

Hanmiao Shen

Mentor: Klara Buysse

June 15, 2017

TABLE OF CONTENT

INTRODUCTION

The purpose of this research project is to calculate the Solvency II ratio of a small insurance company which sells single premium whole life insurance contracts and invests in government bonds and stocks. Since the insurance company is selling insurance contracts with a guaranteed death benefit to the policyholders, it is important for the company to set aside enough money in order to be able to pay back the guarantee it made to the insureds. Therefore, Solvency II regulation is established in order to make sure that the company has enough ability to pay back his liabilities (e.g. death benefit) and thus, Solvency II is protecting policyholders from losing their money from the insurance company.

The main goal of Solvency II is to find the Solvency II ratio, which reflects whether an insurance company is in a healthy condition. Meaning for our insurance company, that it will be able to guarantee the death benefit he promised to his policyholders. In this section, we will discuss the general idea of Solvency II. To calculate the Solvency II ratio, we should have knowledge about two subjects: Own Funds and Solvency Capital Requirement (SCR).

$$Solvency\ II\ ratio = \frac{Own\ Funds}{SCR}$$

Firstly, Own Funds are the own equity of the company. It equals the total assets minus the total liabilities. The total liabilities in our example is the technical provisions which includes the liabilities plus the risk margin. Overall, Own Funds are the total assets minus total liabilities and risk margin.

$$Own\ Funds = Total\ Assets - (Total\ Liabilities + Risk\ Margin)$$

Secondly, SCR is the amount of money that the company needs to reserve to compensate for Market Risk, Life Underwriting Risk and Operational Risk. In our simplified example we will only look at Market and Life Underwriting Risks. The SCR is a Value at Risk measure based on a 99.5% confidence interval of the variation over one year of the amount of the own funds.
In our simplified Solvency II balance sheet, the market risk indicates only the risk caused by the change of interest rates and the change of stock prices.
For interest rate risk there are two possibilities: interest rates can increase or decrease. The effect of these changes in interest rates will be double, namely both assets and liabilities will be impacted. When interest rate increases, the best estimate, calculated as net present value (NPV) in our example, of assets and liabilities, will decrease and vice versa.
For Equity Risk, the impact is caused by a decrease in stock value.
On the other hand, in this project, the Life Underwriting Risk indicates the risk caused by the change of mortality and longevity rate. Mortality rate increase, will increase the death benefit

of policyholders, which means the best estimate of liabilities will also increase. Longevity rate increase has the opposite effect on the best estimate of liabilities, because less people die.

Hence, the SCR can be divided into two parts, market SCR and Life Underwriting SCR. From these two SCRs we can calculate the total SCR and this by means of a correlation matrix. This correlation matrix takes into account the diversification effect between the different risks. It assumes that all the risks don't happen at the same time. The formula to find the total SCR is presented below.

$$SCR = \sqrt{\sum_{i,j} CorrNL_{(i,j)} \cdot SCR_i \cdot SCR_j}$$

The purpose of this report is to give an overview of what our research group has done in the area of Solvency II, which regulates the insurance companies in Europe. This report first introduces the history of Solvency II. Then, it describes some important concepts from Solvency II which will be used in our calculation unfolded later in this report. After that, it introduces a simplified method to calculate Risk Margin. Next, the structure of the mock company and the way we determined the number of bonds and life insurance contracts are explained. After that, a brief preview of the content of the EXCEL file is given, which includes our calculations of SCR and Risk Margin. The calculations for Life Underwriting and Market SCRs, and the way to pick the most appropriate combined SCR are elucidated afterward. Followed by the explanation of the Risk Margin calculation. Then, we calculate the Solvency II ratio from Own Funds and Risk Margin. Finally, we calculate the Solvency II ratio from September to December and investigate the trend of changes of Solvency II ratio.

HISTORY OF SOLVENCY II

The concept of solvency margin requirement was established in 1973 under the First Non-Life Directive (73/239/EEC) and in 1979 under the First Life Directive (79/267/EEC). [Oliver] After several research projects from the European Commission (EC), KPMG and Sharma, Solvency I directives were published and applied in the insurance companies in 2002. [InsuranceERM] Then in 2004 and 2005, the EC issued three waves of calls for advice to the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS). [Oliver] The advice showed that the Solvency I directives didn't include current levels of prudence margin in provisions and couldn't reflect actual level of risk and diversification.

To solve the problems in Solvency I, CEIOPS published reports on four quantitative impact studies from 2005 to 2008 and EC adopts Solvency II in 10 July 2007. In 2008, the concept of solvency capital requirement (SCR) was proposed, which is a key concept in Solvency II. After several adjustments, the text of the Solvency II directive was published in the official journal

in Nov 2009 and Delegated legislation was published in the official journal on 17 Jan 2015. On 1 Jan 2016, Solvency II went live. [InsuranceERM]

Overall, Solvency II is a relatively new concept to manage the risks. The aims of solvency II are to build up a better way to calculate the risks in the insurance companies, to build up a better managed insurance industry which can perform better on spreading risks, and therefore to protect insureds from the default of insurance companies. Since insurance companies sell the guarantee to clients, the failure of risk management will not only destroy an insurance company's business but also damage the health of an economic society. The existence of solvency II will guarantee insurance companies to operate safely and healthily under almost any economy environment, even under the financial crisis. To better achieve this goal, the concept in Solvency II might devise in the future in correspondence of the change in companies' structure and diversified risks.

KEY CONCEPTS FROM SOLVENCY II

The concepts introduced in this section do not include all the concepts that are mentioned in the Solvency II regulation but include only what is needed for our small insurance company. The final goal is to calculate the Solvency II Ratio of our "company". This means that we will need the following important concepts:

- Own Funds
- Solvency Capital Requirement (SCR)

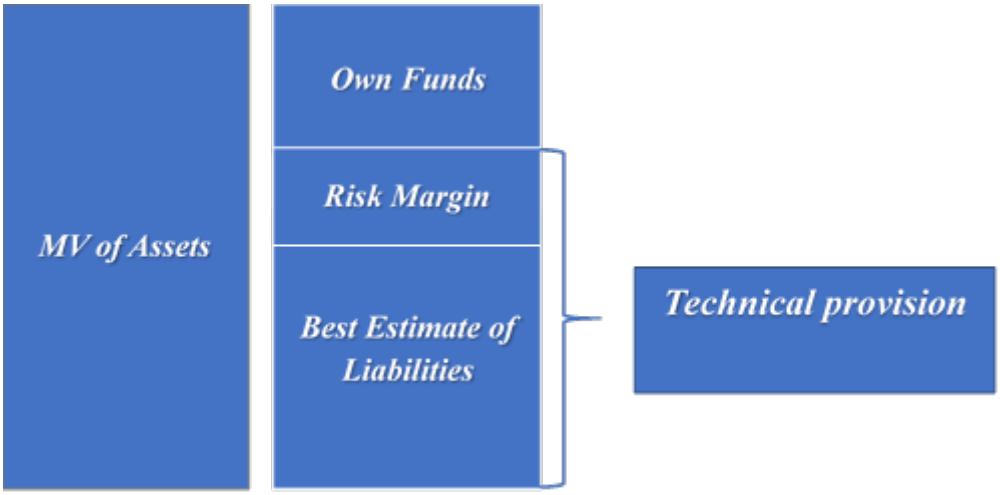
From the Delegated Legislation, we know that the Solvency II Ratio is equal to:

$$Solvency\ II\ Ratio = \frac{Own\ Funds}{SCR}$$

1. OWN FUNDS

The formula above indicates that we need to calculate the Own Funds value of our company. This is the amount of money that represents the company's own capital, which is the surplus capital that is obtained when total liabilities and risk margin are deducted from the total assets. With this amount of money, the company can ensure that his likelihood of being ruined during the year is no more than 1 in 200.

In the following balance sheet, one can find the relationship between Own Funds, Risk Margin, Assets and Liabilities. Everything is valuated at market value (MV). The concept of Market value will be explained in the next section.



As one can see from the balance sheet, we have the relation that:

$$\begin{aligned} Own\ Funds &= Total\ Assets - (Total\ Liabilities + Risk\ Margin) \\ &= Total\ Assets - Technical\ Provisions \end{aligned}$$

2. MV ASSETS

In Solvency II, assets must be valued at the amount for which they could be exchanged in the market between two different parties. The quoted market price is what we should use for this part. However, when the quoted market price of an asset is not available, we can use an appropriate valuation model to find an estimated price. When we use a valuation model to produce future financial market parameters, the following assumptions should hold. These assumptions are from the Delegated Legislation:

"(a) it generates asset prices that are consistent with asset prices observed in financial markets;

(b) it assumes no arbitrage opportunity;

(c) the calibration of the parameters and scenarios is consistent with the relevant risk-free interest rate term structure used to calculate the best estimate as referred to in Article 77(2) of Directive 2009/138/EC."

In our company we invest in two types of assets: government bonds and stocks. For the government bonds we decided to use net present value of all its coupons and its redemption

value in the future as the market value. For the stocks we looked at the quoted values of that period.

3. TECHNICAL PROVISIONS

As mentioned above, technical provisions are the total liabilities of the company with the Risk Margin included. This is equivalent to the formula below.

$$Technical\ Provisions = Best\ Estimate\ of\ Liabilities + Risk\ Margin$$

4. BEST ESTIMATE OF LIABILITIES

Because we need to use best estimate of liabilities to find technical provisions, the next actuarial term we want to discuss is best estimate of liabilities. Since the liabilities of our company are the cash flow of death benefits that we need to pay to the policyholders, we will use the NPV of liabilities as its best estimate.

To reduce controversy on what is seen as an insurance contact, the solvency II regulation defines the conditions that an insurance contract needs to satisfy:

"For the calculation of the best estimate and the risk margin of technical provisions, insurance and reinsurance undertakings shall recognise an insurance or reinsurance obligation at the date the undertaking becomes a party to the contract that gives rise to the obligation or the date the insurance or reinsurance cover begins, whichever date occurs earlier. Insurance and reinsurance undertakings shall only recognise the obligations within the boundary of the contract. Insurance and reinsurance undertakings shall derecognise an insurance or reinsurance obligation only when it is extinguished, discharged, cancelled or expires."

Also, the Solvency II regulation defines the boundary of insurance or reinsurance contracts.

"Obligations which relate to insurance or reinsurance cover provided by the undertaking after any of the following dates do not belong to the contract, unless the undertaking can compel the policyholder to pay the premium for those obligations:

(a) the future date where the insurance or reinsurance undertaking has a unilateral right to terminate the contract;

(b) the future date where the insurance or reinsurance undertaking has a unilateral right to reject premiums payable under the contract;

(c) the future date where the insurance or reinsurance undertaking has a unilateral right to amend the premiums or the benefits payable under the contract in such a way that the premiums fully reflect the risks."

Briefly speaking, if the insurance or reinsurance company wants to terminate the contract, or reject the premiums for this contract, or amend the premiums or the benefits, these cash flows will not belong to the contract any more.

A famous example of this is a Universal Life contract with regular premiums. These contracts have guaranteed rates on paid premiums but not on future premiums, meaning that the company can adapt the guaranteed rates in case of difficulties. The concept of contract boundaries than says that the company cannot consider future premiums in his cash flow projections. In our small insurance company, we have a single premium contract with no guaranteed rates on paid premiums and thus they are within the boundaries of the contract.

In order to project the liabilities by means of a cash flow model, the company needs to set assumptions about mortality tables, lapse rates, expenses, …. For this Solvency II requires the following:

"Assumptions shall only be considered to be realistic for the purposes of Article 77(2) of Directive 2009/138/EC where they meet all of the following conditions:

(a) insurance and reinsurance undertakings are able to explain and justify each of the assumptions used, taking into account the significance of the assumption, the uncertainty involved in the assumption as well as relevant alternative assumptions;

(b) the circumstances under which the assumptions would be considered false can be clearly identified;

(c) unless otherwise provided in this Chapter, the assumptions are based on the characteristics of the portfolio of insurance and reinsurance obligations, where possible regardless of the insurance or reinsurance undertaking holding the portfolio;

(d) insurance and reinsurance undertakings use the assumptions consistently over time and within homogeneous risk groups and lines of business, without arbitrary changes;

(e) the assumptions adequately reflect any uncertainty underlying the cash flows.

Besides this, the solvency II regulation has also some guidelines on the assumptions on future management actions.

"1. Assumptions on future management actions shall only be considered to be realistic for the purposes of Article 77(2) of Directive 2009/138/EC where they meet all of the following conditions:

(a) the assumptions on future management actions are determined in an objective manner;

(b) assumed future management actions are consistent with the insurance or reinsurance undertaking's current business practice and business strategy, including the use of risk-mitigation techniques; where there is sufficient evidence that the undertaking will change its practices or strategy, the assumed future management actions are consistent with the changed practices or strategy;

(c) assumed future management actions are consistent with each other;

(d) assumed future management actions are not contrary to any obligations towards policy holders and beneficiaries or to legal requirements applicable to the undertaking;

(e) assumed future management actions take account of any public indications by the insurance or reinsurance undertaking as to the actions that it would expect to take or not take.

2. Assumptions about future management actions shall be realistic and include all of the following:

(i) a comparison of assumed future management actions with management actions taken previously by the insurance or reinsurance undertaking;

(ii) a comparison of future management actions taken into account in the current and in the past calculations of the best estimate;

(iii) an assessment of the impact of changes in the assumptions on future management actions on the value of the technical provisions."

As a result, future management actions, such as changing the guaranteed rate of the contract, should be consistent with market changes and other companies. The future management actions have to protect the policyholders.

In Solvency II, liabilities must also be valued at their market value. This is more difficult, because liabilities are not traded on a market. Consequently, they should be estimated by means

of a Cash Flow model designed by companies in an attempt to calculate the estimated market value of liabilities.

Best Estimate (BE) is another important concept frequently used to calculate SCR, Risk Margin, and other values. For our single premium whole life insurance product, we use net present value (NPV) as its best estimate which represents the value of this product at t=0.

5. RISK MARGIN (RM)

Risk Margin is another really crucial value to be used to calculate the Solvency II ratio. It is the extra amount of money that the insurer has to set aside in order to have enough money for the liabilities in case the best estimate assumptions were not correctly estimated. The risk margin can also be seen as the extra cost that transfers obligations to a third party. To calculate the risk margin, we use the cost of capital method and the Life Underwriting SCRs (see section 2 under Solvency Capital Requirement on page 10). To calculate these values, we need the Best Estimate of Liability and the Life Underwriting Solvency Capital Requirement.

6. COST-OF-CAPITAL (COC)

The last concept we want to explain is the Cost-of-Capital rate (COC). This is the cost of funds used for financing a business. In our calculation, we assume a 6% CoC rate for the calculation of the risk margin. This 6% is imposed by EIOPA. Once we get the Solvency Capital requirement and some important Best Estimate values, we will use the Cost of Capital to combine these values to get the total Risk margin.

PROPORTIONAL METHOD FOR SIMPLIFIED CALCULATION OF RISK MARGIN:

The official method to calculate the Risk Margin requires to calculate future SCRs, which is hardly possible for almost all companies. In our research, we attempt to calculate Risk Margin using the most common used simplified method, proportional approach.

The standard formula for Risk Margin is the following:

$$RM = CoC \times \sum_{t=0}^{n} \frac{SCR_t}{(1+r_f)^{t+1}}$$

CoC: given as 6%
r_f: risk-free rate for t+1 year maturity

As stated above, the SCR_t in the future is unknown and is difficult to predict for most companies. As a result, the proportional approach approximates them using the following formula:

$$SCR_t = BE_t \times \left(\frac{SCR_0}{BE_0}\right)$$

Therefore, our first goal is to calculate the SCR at time 0, and the best estimate of liabilities at time 0. By means of the cash flow model we can predict the values of the best estimate of liabilities at future times steps t. Then we calculate the SCR for each year in the future by using the ratio of the best estimate of liabilities at time step t dividing by the best estimate at times step 0. Finally, we can calculate the Risk Margin with the formula above.

SOLVENCY CAPITAL REQUIREMENT

Solvency Capital Requirement (SCR) is the amount of money that insurance and reinsurance undertakings are required to hold. The SCR is based on a Value-at-Risk measure calibrated to a 99.5% confidence level over a 1-year time horizon. This means that the SCR protects the company against a 1 in 200 event and this for 1 year.

In our example, it should be the amount that can ensure that all quantifiable risks are considered, meaning Life Underwriting and Market Risks. For our company, Life Underwriting Risk includes mortality risk and longevity risk, the Market Risk part on the other hand includes interest risk related to the bonds in our asset account and equity risk.

Solvency II ratio is the ratio that evaluates the ability to face a crisis. A Solvency II ratio larger than 100% means that the own capital is larger than the capital requirement, which shows that

the company has the basic security to survive terrible financial conditions. Also, this ratio is an important index to evaluate the financial health of the company. It is best to exceed 150%, which can demonstrate a relatively healthy financial status of an insurance company.

1. MARKET RISKS

Market risk is the risk that the company will have losses as a consequence of the overall performance of financial markets. The market risks for our company can be divided into two parts. The first one is interest rate risk, which is caused by the change of interest rate. The other one is equity risk, which is caused by the change of equity market.

Since our company sells whole life insurance contracts and invests in government bonds, both the assets and the liabilities will depend on interest rate risk due to the principle of Market Value valuation. The Market Value of the government bonds and the Best Estimate of Liabilities is determined by discounting all the cash flows back to t=0 using the interest rate term structure (September curve) offered by EIOPA.
If the interest rate increases, the market value of assets and liabilities will decrease, and vice versa. Hence, the assets and liabilities of our company will be influenced by the interest rate, while the interest rate is determined by the financial market. This is why there exists market risk.

To quantify this risk, we use the increased and decreased interest rate term structure (September curve) which is provided by EIOPA. This will give us the new market value of assets and liabilities under these stresses.

Also, we calculated the market value of assets and liabilities using the regular interest rate. Then we do the following:

$$SCR_{interest\ down} = (Assets_{interest\ down} - Assets_{regular}) - (Liabilities_{interest\ down} - Liabilities_{regular})$$
$$SCR_{interest\ up} = (Assets_{interest\ up} - Assets_{regular}) - (Liabilities_{interest\ up} - Liabilities_{regular})$$

We calculate the differences between the market values using regular interest rate and the market value using shocked interest rate. There will be two results, one for the interest rate up scenario and one for the interest rate down scenario. We choose the larger one as the market risk SCR. The calculation details will be explained in later sections.

Since our company also invests in stocks, where the price is affected by the change observed in the equity market, we need to take into account equity risk into the Market Risk calculations. The detailed calculation will also be explained in later sections.

2. LIFE UNDERWRITING RISKS

For the company in this project, Life Underwriting risk is the risk that the company will have losses because of the change in mortality rates. Since our company sells whole life insurance contracts and the contract pays a death benefit of 20,000 at the end of the year that the policyholder dies, the best estimate of liabilities will depend on the mortality rates of the policyholders. To find this risk, we increase the mortality rate by 15% (see Solvency regulation) and find the best estimate of liabilities. Then, we calculate the difference between this shocked best estimate and the regular one. This difference amount indicates how much money our company needs to reserve for Life Underwriting Risk.

$$SCR_{mortality} = BE_{mortality} - BE_{regular}$$

Also, we decrease the mortality rate by 20% (see Solvency regulation) and repeat the steps to find its best estimate and the difference between the best estimates.

$$SCR_{longevity} = BE_{longevity} - BE_{regular}$$

Finally, we will use a correlation matrix to combine these two SCR to calculate the total Life Underwriting SCR.

THE "MOCK COMPANY"

Assume that we are a small European insurance company regulated by Solvency II. This section explains and defines in detail the products and the structure of our mock company. More specific, the type of assets and liabilities that are incorporated in the company.

1. ASSETS

To get familiar with the concepts of Solvency II, we used government bonds and stocks as assets in the balance sheet of the company.

More specific, we will look at US Treasury Bonds. To conduct further calculations, we gathered information of 2-year T-bond, 5-year T-bond, 10-year T-bond, and 30-year T- bond from the website of US Treasury. This information includes coupon rates, current price, par value, maturity, and etc.
We also looked at the US stock market and chose to invest in APPLE and GOOGLE stock each for 3 shares.

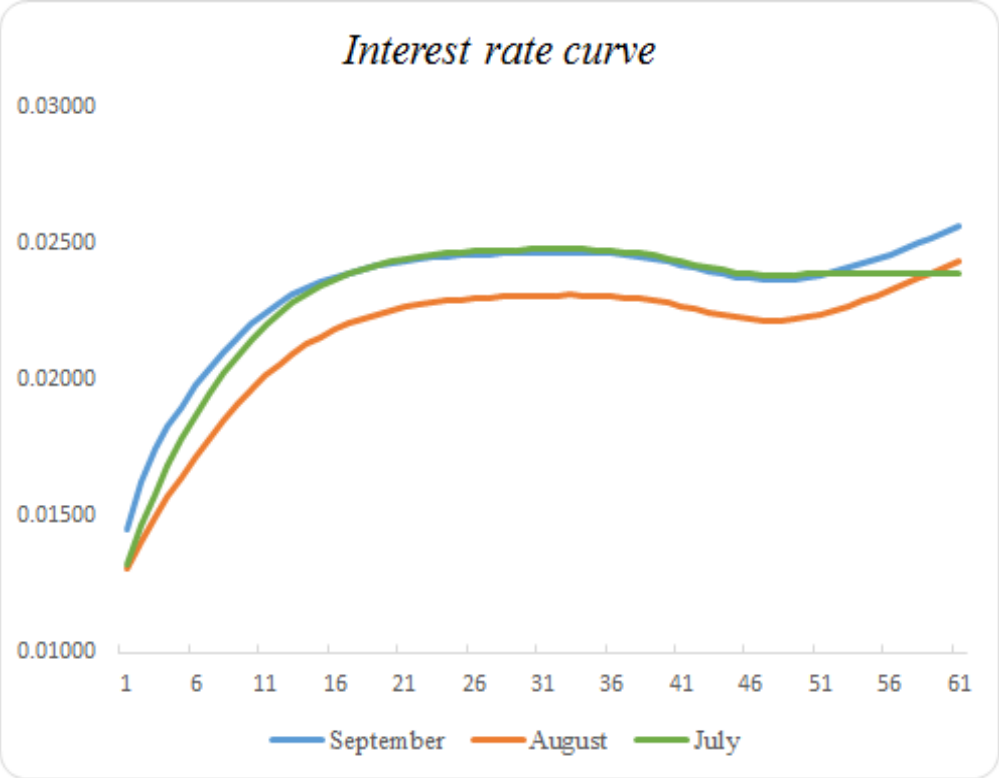
2. LIABILITIES

For the liability part of the company, we will look at whole life insurance contracts for people of the same age.

The contract pays a death benefit of 20,000 at the end of the year if the person dies. The policyholders pay a single premium at the beginning of the contract. The amount of the premium is the net present value of the possible future death benefit with survival model taken into consideration.

For the mortality table, we have chosen the life table for the total population in United States in year 2014 from the National Vital Statistics System (NVSS).

The interest rate term structure is officially provided by the European Insurance and Occupational Pensions Authority (EIOPA). These are updated each month. For the mock company, we will use the interest rate term structure of September 2017. In the following graph, we compare the interest rate curve of September 2017 with July and August. The graph of the curve is below.



3. THE BALANCE SHEET

In order to have a balance sheet that is in equilibrium we have to decide in how many bonds we will invest.

The way we will determine the number of bonds is by calculating and tracking duration mismatch. An insurance company definitely wants to minimize the duration mismatch between assets and liabilities. If there is a big duration gap between assets and liabilities, the risk exists that the company will have difficulties to pay back the policyholders in the future, because the assets don't provide enough cash flows to pay off the liabilities.

We calculated the durations of the 2-year, 5-year, 10-year, and 30-year US Treasury bonds, and the duration of the whole life insurance contract. The duration of 30-year US T-bond matched the duration of the life contract best. Therefore, we picked the 30-year T-bond.

Also, we added another 30-year T-bond after the expiration of the first 30-year T-bond, so that the coupons from the overall 60 years of bonds can be used to pay for the death benefits.

Additionally, we set the number of bonds to 25, in order to make the amount of assets similar to the amount of liabilities.

In the end, we calculated the duration and convexity of our assets (T-bonds) and liabilities (death benefit of whole life insurance contract). The results show that the duration of assets is 30.45, and the duration of liabilities is 33.98. The duration of asset and liability are similar, which shows that the cash flows of liabilities can be paid by the assets. Also, the duration of assets is smaller than duration of liability meaning that the assets takes less time to get the money back than the liabilities. We will use SCR to compensate for this mismatch of duration.

Accordingly, we can summarize and construct the following balance sheet:

CALCULATION OF THE SII RATIO OF THE "MOCK COMPANY"

We used MS EXCEL to conduct our calculations of BE, SCR, and RM. In this section, we explain the different calculations conducted in several sheets of the EXCEL file.

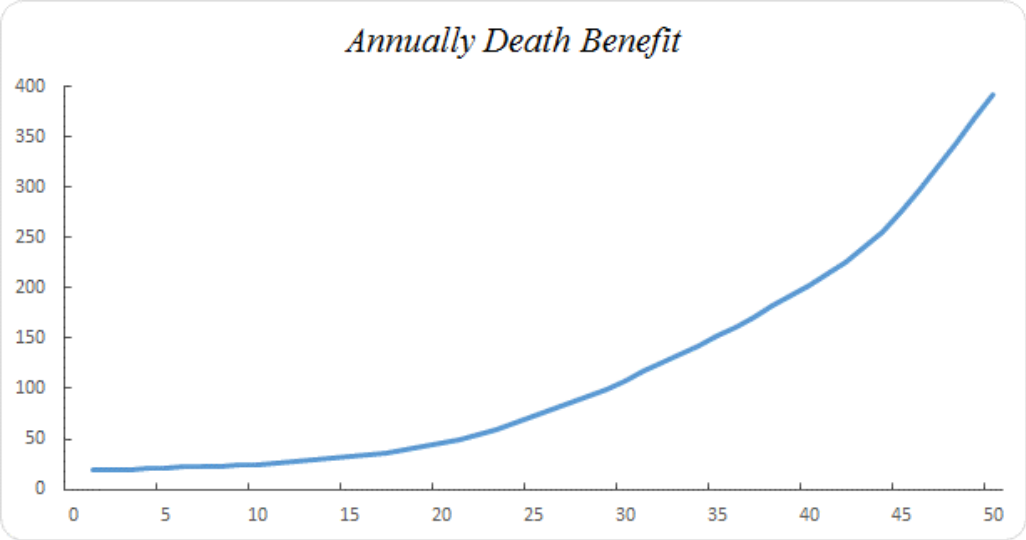
1. LIABILITIES

The first sheet is about the liabilities, which contains the single premium whole life insurance contract with death benefits.

We have the following assumptions:

- Single premium of 2,474.26
- Death benefit of 20,000
- Mortality table from NVSS (Year 2014 life table)
- Interest rate curve from EIOPA (September curve)

We assume that the policyholders have paid their premium before t=0. By means of the mortality table we calculate the death benefit of each time step, which gives us the total cash flow of the contract. Here is the graph of death benefit for each year.



These cash flows are then discounted with the discount curve. Created with the following formula.

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}$$

This gives us a total BE of

Best Estimate of Liabilities = 2,474.26

We refer to the worksheet "mortality analysis" in which one can find the probability of dying each year, from year 0 to year 50. Then, we calculate the probability that a person dies at year t (from 0 to 50). Using this survival model, we find the cash flow (from year 1 to year 50) by multiplying the death benefits with the probability of dying for each year. After that, we use the interest rate term structure from the worksheet "interest curve" to calculate the discount factors. Since the interest rates provided are spot rates, we can calculate discount factors by

using the formula $(1+i)^{-t}$. With these discount factors, we find the BE (NPV) by summing up all the values of the cash flows at time 0.

This gives us the best estimate of liabilities, but as we saw earlier we also need to calculate the value of the liabilities for the mortality risk submodule, longevity risk submodule, interest rate up risk submodule, and interest rate down risk submodule. This can be found in the right part of the sheet "Liability". The longevity risk and mortality risk submodules will be used to calculate the life part SCR of liabilities and the interest rate risk submodule will be used to calculate the market part SCR of liabilities. For each risk, we calculate the Best Estimate (NPV) of the death benefits, and then we find the difference of the BE in the shocked scenario with the BE in the regular scenario in order to find the SCR for each submodule.

For the calculation of the interest rate risk SCR we need the change in amount of liabilities together with the change in amount of asset. The summary of these values is in the yellow table on the very left side of the worksheet "Liability".

2. ASSETS

There are two main sections in assets:

- government bonds,
- stocks

The second sheet "Asset" is about the bond section. There are three main sections in this sheet:

- regular interest rate (no-shock),
- increased interest rate and
- decreased interest rate.

In the no-shock section, we calculate the BE (NPV) of 1 share of the 30-yr US T-bond. In the interest rate up and interest rate down sections, we calculate the NPV of "30+30" T-bond portfolio from time 0 to time 60, using the interest rate up and interest rate down term structures from EIOPA. We also calculate the duration and convexity of the "30+30" T-bond portfolio, which are 30.45 and 387.39.

The total value of bonds is equal to:

$$\textit{Total Value of Bond} = 3{,}898.22$$

Our company also invests in APPLE stocks for three shares and GOOGLE stocks for three shares. The third sheet "BE and SCR" includes this information. Since this solvency report is based on September, we used the month price for the stocks. In that month, the price of APPLE is 167.75, and the price of GOOGLE stock is 180.06.

The total value of stock is equal to:

$$Total\ Value\ of\ stock = (167.75 + 180.06) * 3 = 1043.43$$

The total value of assets is equal to:

$$Total\ Value\ of\ Assets = 3898.22 + 1043.43 = 4941.65$$

This gives us the following balance sheet:

We still need to calculate the Risk Margin in order to find the Own Fund value. For this we need to calculate first the Life Underwriting SCRs.

3. LIFE UNDERWRITING SCRS

Since our company only has whole life insurance products with death benefits, the only Life Underwriting SCR involved is mortality risk and longevity risk. This is due to the use of the Mortality table. We assume there are no expenses or surrender options in the contract.

The total life SCR for the company is calculated as follows:

$$SCR_{life} = \sqrt{\sum_{i,j} CorrL_{(i,j)} \cdot SCR_i \cdot SCR_j}$$

where i and j represent the different risks. This formula is published in the Delegated Acts.

For the small insurance company, we assume only mortality or longevity risk. To calculate the SCRs of these two risks, we can first interpret them as the risk that the chosen mortality table is wrong, which means that the mortality rates are higher or lower than expected. Thus, the SCRs of these two risks are set to compensate for differences in the true mortality rates and the estimated/provided mortality rates.

The table below is the correlation coefficients between these two risks, also published in the Delegated Acts.

i j	Mortality	Longevity
Mortality	1	-0.25
Longevity	-0.25	1

To calculate the total life SCR, we have to calculate first the mortality and longevity risk separately.

In the delegated Acts, one can find the different shocks an insurance company has to apply in order to calculate these risks.

For mortality we have the following (Article 137)

"The capital requirement for mortality risk shall be equal to the loss in basic own funds of insurance and reinsurance undertakings that would result from an instantaneous permanent increase of 15 % in the mortality rates used for the calculation of technical provisions"

We multiply all mortality rates by 115%. Then, we do the same calculation to get the BE (NPV) as in the first part (no-shock).

This gives us a BE of 2,789.94.

This means that the SCR for mortality is equal to:

$$SCR_{mortality} = BE_{mortality} - BE_{regular}$$
$$SCR_{mortality} = 2{,}789.94\text{-}2{,}474.26 = 315.68$$

This means that the SCR for mortality risk is equal to the difference of the BE with increased mortality rates and the BE_0 with regular mortality rates. And this by assuming the regular interest rate curve.

For longevity risk, we have the following (Article 138)

"The capital requirement for longevity risk referred to in Article 105(3)(b) of Directive 2009/138/EC shall be equal to the loss in basic own funds of insurance and reinsurance undertakings that would result from an instantaneous permanent decrease of 20 % in the mortality rates used for the calculation of technical provisions."

For this scenario we have to decrease the mortality rates. Similarly, to the previous step we decrease the mortality rates by 20%, and this by multiplying all the mortality rates by 80%. Then, we do the same calculation to get BE (NPV) for the longevity submodule.

We find the longevity risk SCR submodule with decreased mortality rate by subtracting the BE_0 with regular mortality rates from the BE_0 with decreased mortality rate, and this by assuming the regular interest rate curve.

The calculation gives us：

$$BE_{longevity} = 2{,}032.64$$

This means that the SCR for mortality is equal to:

$$SCR_{longevity} = BE_{longevity} - BE_{regular}$$
$$SCR_{longevity} = 2032.64\text{-}2{,}474.26 = -441.62$$

A negative value means that this company doesn't have a longevity risk. Since the amount of money reserved cannot be negative, we take this number as 0.

Then we have to calculate the total life SCR, by means of the following formula.

$$SCR_{life} = \sqrt{\sum_{i,j} CorrNL_{(i,j)} \cdot SCR_i \cdot SCR_j}$$

$$SCR_{life} = \sqrt{SCR_{mortality}^2 + SCR_{longevity}^2 + 2 \times (-0.25) \times SCR_{mortality} \times SCR_{longevity}}$$

The correlation between mortality risk and longevity risk is -0.25 from the table above in this section. The risk for longevity risk is zero which means that the formula can be reduced to:

$$SCR_{life} = \sqrt{SCR_{mortality}^2} = SCR_{mortality}$$

Finally, we calculate the total life SCR as:

$$SCR_{life} = 315.68$$

4. MARKET SCRS

The total market SCR for the company is calculated as follows:

$$SCR_{market} = \sqrt{\sum_{i,j} CorrL_{(i,j)} \cdot SCR_i \cdot SCR_j}$$

where i and j represent the different risks. This is the formula published in the Delegated Acts. Since we have stocks and bonds in our company's portfolio. Interest rate risk and equity risk are the Market Risks for our company. In this case, the i and j represent interest rate risk and equity risk.

In the table below one can find the correlation coefficients between these two risks, also published in the Delegated Acts.

i j	Interest rate	Equity
Interest rate	1	0.5
Equity	0.5	1

To calculate the total Market SCR, we have to calculate the interest rate and equity risk separately.

First, we want to explain how to find the SCR for interest rate risk. If interest rate changes, the value of the Best Estimate of liabilities and government bonds will change correspondingly. We will use these two amounts of change in asset and liability, respectively, to calculate the $SCR_{interest}$ in the interest risk submodule. The following relations will be used in our later calculations.

$$\triangle assets = Assets_{interest\ shock} - Assets_{regular}$$

$$\triangle liabilities = Liabilities_{interest\ shock} - Liabilites_{regular}$$

For Solvency II there are two kind of interest rate risks:

- Interest rate up risk
- Interest rate down risk

For increased interest rate we have the following explanation (Article 166)

"The capital requirement for the risk of an increase in the term structure of interest rates for a given currency shall be equal to the loss in the basic own funds that would result from an instantaneous increase in basic risk-free interest rates for that currency at different maturities in accordance with the following table:"

For decreased interest rate we have the following explanation (Article 167)

"The capital requirement for the risk of a decrease in the term structure of interest rates for a given currency shall be equal to the loss in the basic own funds that would result from an instantaneous decrease in basic risk-free interest rates for that currency at different maturities in accordance with the following table:"

In our project, specifically, we will use the shock tables provided by EIOPA. This includes the basic shock percentage. To calculate the increased and decreased interest rates, we should follow the steps in the two paragraphs below.

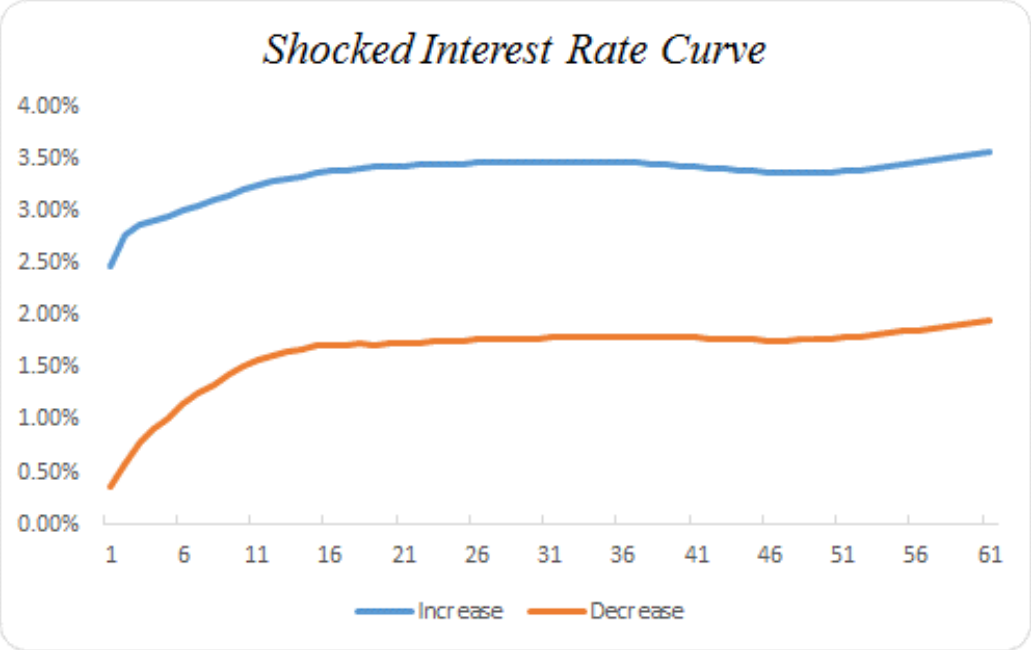
The interest rate shock table provides the shock rates for each year. To calculate the increased interest rate, the first step is to multiply the shock rate from the table by the regular interest rate

for each year. Then, we pick the larger one between this amount and 1% as the increase percentage to be added on the corresponding year's regular interest rate.

Likewise, to find the decreased interest rate, we first multiply the shock rate in table from EIOPA by the regular interest rate for each year. If the regular interest rate of a year is below 0, we use the original one, and otherwise we subtract the calculated value in the first step from the regular interest rate.

The details and formulas to calculate the increased and decreased interest rates can be found in the "interest curve" sheet of our EXCEL file.

After calculating the decreased and increased interest rate term structure, we obtain the following graph of shocked interest rate curves:



With these curves, we have the tool to calculate the change of assets and liabilities. Still we will use the concept of Best Estimate, which is the NPV of the liability cash flows in our example. To do this, we use the following formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}$$

To begin with, we calculate the changes in liabilities first.

We find that the BE of interest rate up is:

$$BE_{interest\ up} = 1{,}797.39$$

and the BE of interest rate down is:

$$BE_{interest\ down} = 3{,}018.63$$

Hence, the difference between the BE_0 with interest rate up of liabilities and BE_0 with regular interest rate of liabilities is

$$\triangle liability_{interest\ up} = BE_{interest\ up} - BE_{regular}$$

$$\triangle liability_{interest\ up} = 1{,}797.39 - 2{,}474.26 = -676.87$$

The difference between the BE_0 with interest rate down and BE_0 with regular interest rate is

$$\triangle liability_{interest\ down} = BE_{interest\ down} - BE_{regular}$$

$$\triangle liability_{interest\ down} = 3{,}018.63 - 2{,}474.26 = 544.37$$

After calculating the changes in liabilities in the two scenarios of interest rate shocks, we conduct similar calculations in our asset account.

Here we find that the BE of interest rate up is

$$BE_{interest\ up} = 2{,}948.40$$

and the BE of interest rate down is

$$BE_{interest\ down} = 4{,}675.46$$

Hence, the difference between the BE_0 with interest rate up of assets and BE_0 with regular interest rate of assets is

$$\triangle asset_{interest\ up} = BE_{interest\ up} - BE_{regular}$$

$$\triangle asset_{interest\ up} = 2{,}948.40 - 3{,}898.22 = -949.82$$

The difference between the BE_0 with interest rate down and BE_0 with regular interest rate is

$$\triangle asset_{interest\ down} = BE_{interest\ down} - BE_{regular}$$

$$\triangle asset_{interest\ down} = 4{,}675.46 - 3{,}898.22 = 777.24$$

Therefore, the SCR for interest rate up is equal to:

$$SCR_{interest\ up} = \triangle asset_{interest\ up} - \triangle liability_{interest\ up}$$

$$SCR_{interest\ up} = -949.82 - (-676.87) = -272.95$$

The SCR for interest rate down is equal to:

$$SCR_{interest\ down} = \triangle asset_{interest\ down} - \triangle liability_{interest\ down}$$

$$SCR_{interest\ down} = 777.24 - 544.37 = 232.87$$

We choose the larger one between the $SCR_{interest\ up}$ and $SCR_{interest\ down}$ as the final interest rate SCR. Thus, the interest rate SCR of our company is 232.87.

Secondly, we explain the process to find equity SCR. This SCR is caused by the changes in the equity market. When the equity market performs poorly, the value of the stocks will decrease, and when equity market performs well, the value of the stocks will increase. The only thing we care about as a risk, is the decreasing price. Therefore, for equity risk, we only have equity down risk.

For decreased equity value we have the following explanation (Article 169)

"an instantaneous decrease equal to the sum of 39 % and the symmetric adjustment as referred to in Article 172 of this Regulation, in the value of type 1 equities other than those referred to in point (a)"

According to the Regulation, the price after shock is 39% of the regular price. Then the equity SCR is equal to:

$$stock_{equity\ down} = stock_{regular} * 0.39$$
$$SCR_{equity} = stock_{regular} - stock_{equity\ down}$$

From the formulas shown above, we find that the equity SCR is equal to 636.49.

Now we can calculate the total market SCR, by means of the following formula:

$$SCR_{market} = \sqrt{\sum_{i,j} CorrL_{(i,j)} \cdot SCR_i \cdot SCR_j}$$

$$SCR_{market} = \sqrt{SCR_{interest\ rate}^2 + SCR_{equity}^2 + 2 \times (0.5) \times SCR_{interest\ rate} \times SCR_{equity}}$$

The correlation between interest rate risk and equity risk is 0.5, this can be found in the table at the beginning of this this section.

Finally, we have that the total market SCR is:

$$SCR_{market} = 779.47$$

THE WAY TO FIND THE BASIC SCR

From the calculations explained above, we have a Life Underwriting SCR and a Market SCR.

Finally, we can find the Basic SCR by using the formula below (see Delegated Acts).

$$BasicSCR = \sqrt{\sum_{i,j} Corr_{(i,j)} \cdot SCR_i \cdot SCR_j} + SCR_{int}$$

In our company, we don't have intangible assets, meaning that the SCR intangible (SCR_{int}) is equal to zero. We only have an SCR from the market risk submodule and an SCR from the life risk submodule. From publication of EIOPA, the correlation coefficient of these two is 0.25. The means that we have the following formula:

$$BasicSCR = \sqrt{SCR_{Market}^2 + SCR_{Life}^2 + 2 \times (0.25) \times SCR_{market}.SCR_{Life}} + 0$$

Which gives us a Basic SCR of:

$$SCR_{basic} = 911.18$$

CALCULATION OF RISK MARGIN

First, we calculate the best estimate of liabilities at t=0 (NPV_0). Then the BE_t of liabilities at time is t (NPV at time = t) can be calculated by multiplying the NPV at the last year (t-1) by the interest rate, minus the cash flow at that year t. For example, NPV_1 can be found by multiplying the NPV_0 by the 1+interest rate, and minus the cash flow at t=1. In this way, we can find the Best Estimate of liability from t=0 to t=50.

Since we have found the SCR_0 in the previous section, we can find the SCR_t using the formula below from t=0 to t=50.

$$SCR_t = BE_t \times \left(\frac{SCR_0}{BE_0}\right)$$

Finally, we can find the risk margin using the formula below. The Cost-of-Capital rate (CoC) is equal to 6%.

$$RM = CoC \times \sum_{t=0}^{n} \frac{SCR_t}{(1+r_f)^{t+1}} = 659.26$$

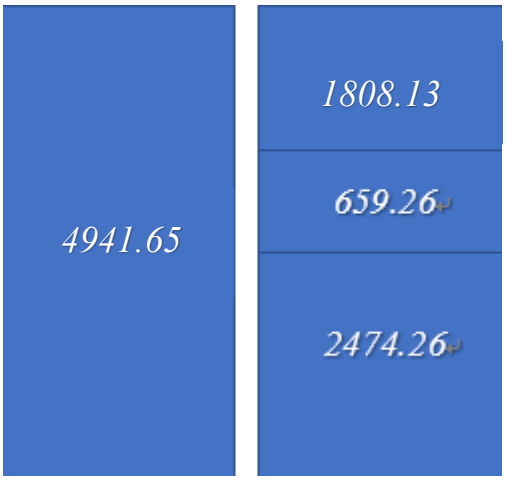
THE BALANCE SHEET

The last thing we can calculate in the balance sheet is the value of the Own Funds. From the section "Introduction" we have the formula:

$$Own\ Funds = Total\ Assets - (Total\ Liabilities + Risk\ Margin)$$

$$Own\ Funds = 4941.65 - (2{,}474.26 + 659.26) = 1808.13$$

Then the solvency II balance sheet of the company if completed:



CALCULATION OF SOLVENCY II RATIO

The solvency II ratio is the most important and final result of this project. All the steps above have served to calculate this final ratio. The formula to find the Solvency II ratio is presented below, which is the Own Funds divided by the Solvency Capital Requirement.

$$Solvency\ II\ ratio = \frac{Own\ Funds}{SCR}$$

$$SII\ ratio = 198.44\%$$

The solvency II ratio of the company is 198.44%, which is larger than 100%. This result shows that we have a healthy company and thus we have enough capital to pay policyholders in difficult scenarios.

CONCLUSION FROM THE CALCULATIONS IN EXCEL

In this Solvency II research project, we are the actuaries of a small insurance company, which sells whole life insurance contracts and invests in government bonds and stocks. The whole life insurance contract has a single premium, and a death benefit of 20,000 payable at the end of the year of death with a maturity of 50 years. We invest in 30-year US Treasury bonds and in APPLE and GOOGLE stocks.

We found the Solvency II ratio for the month of September and we have built up the corresponding Solvency II balance sheet. This means we have calculated the Best Estimate

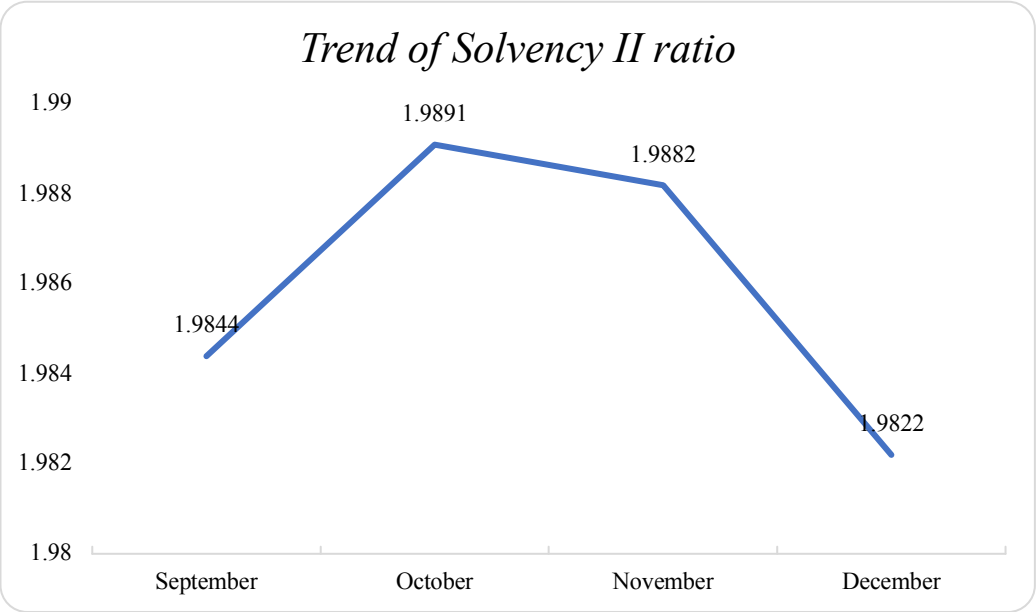
of Liabilities (2,474.26), the Solvency Capital Requirement (315.68) and the Risk Margin (659.26).
The calculations show that our company needs to reserve an extra amount of 659.26 for the liabilities because the best estimate value could be not enough in case of extreme events. Also, the SCR_0 is 315.68 which shows that the company needs to reserve 315.68 to resist extreme market and life risks. The SCR protects the company against a 1 in 200 event and this for 1 year.
The solvency II ratio of our company is 198.44%, which is larger than 150%. Therefore, this solvency II ratio strongly demonstrates the financial health of our company, and it has the ability to give policyholders the guarantee to pay back their liabilities.

THE SOLVENCY II RATIO FOR FOLLOWING MONTHS

After finding the Solvency II ratio for September, we want to look at the change of the Solvency II ratio over different months. Therefore, we will calculate the solvency II ratio from September to December. In the later sections, we will display the results we have found and analyze the movements.

1. THE RESULTS

In this section, we show the trend of the Solvency II ratio over time. From the graph, we can tell that October has the largest solvency II ratio, and December has the lowest solvency II ratio. What's more, the Solvency II ratio increases from September to October, and after that decreases from October to December. In the later sections, we will analyze the differences between these months and find the possible reasons why we obtain these results.



2. ANALYSIS OF THE TREND

Before analyzing the results, we want to understand how interest rate and equity affects the Solvency II ratio. Let's first look at interest rates. Since Solvency II uses the concept of Market Value for both assets and liabilities, interest rate changes will affect both liability and assets. We use the following formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}$$

When interest rates increase, the liabilities and assets will both decrease and vice versa. While interest rates do not only affect liabilities and assets, it will also affect the market and life underwriting SCRs.

Notice that for SCR we look at the difference between assets and liabilities. Hence, the change in SCR due to interest rates is hard to predict. When interest rates increase, the difference between assets and liabilities can increase, decrease or keep the same. This depends on the duration mismatch.

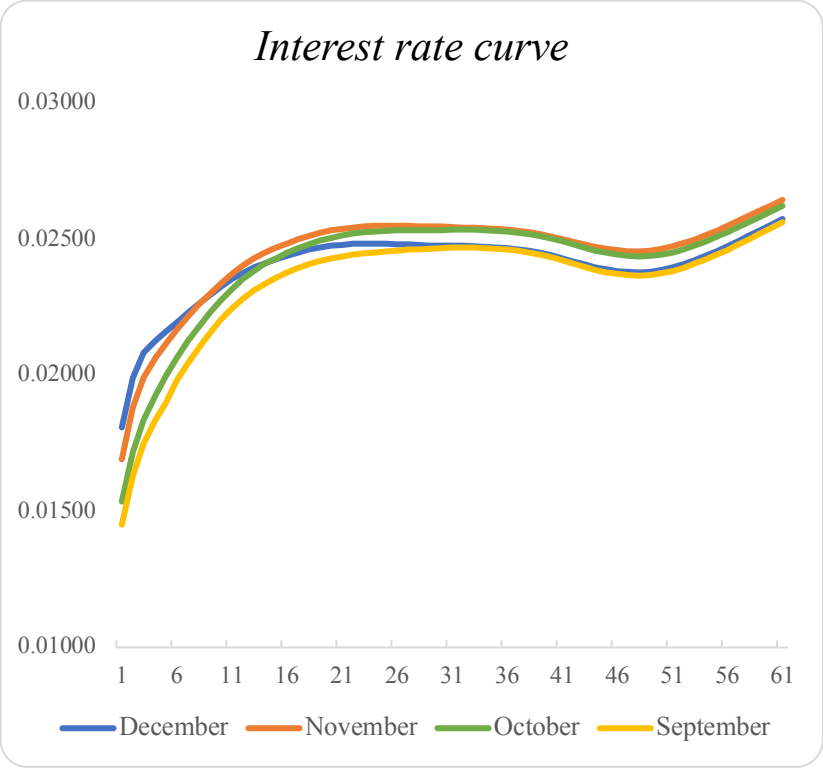
Since the mortality rates and shock rates will not change for these months. The difference between regular mortality rate and shocked mortality rate will not change. The only thing that changes is the interest rate. Therefore, when interest rates increase, the mortality SCR will also decrease. Since the risk margin is calculated by means of mortality SCR, the risk margin will also fluctuate according to mortality SCR.

The last thing is interest rate SCR. As the name states, interest rate SCR will be different when interest rates change. Since interest rate SCR is calculated by means of the difference between

assets and liabilities, it is difficult to tell how it will change. In conclusion, interest rate will affect assets, liabilities, mortality SCR, risk margin and interest rate SCR. Since it is significant sophisticated process to find the Solvency II ratio and all these factors can influence the result, we will analyze this later.

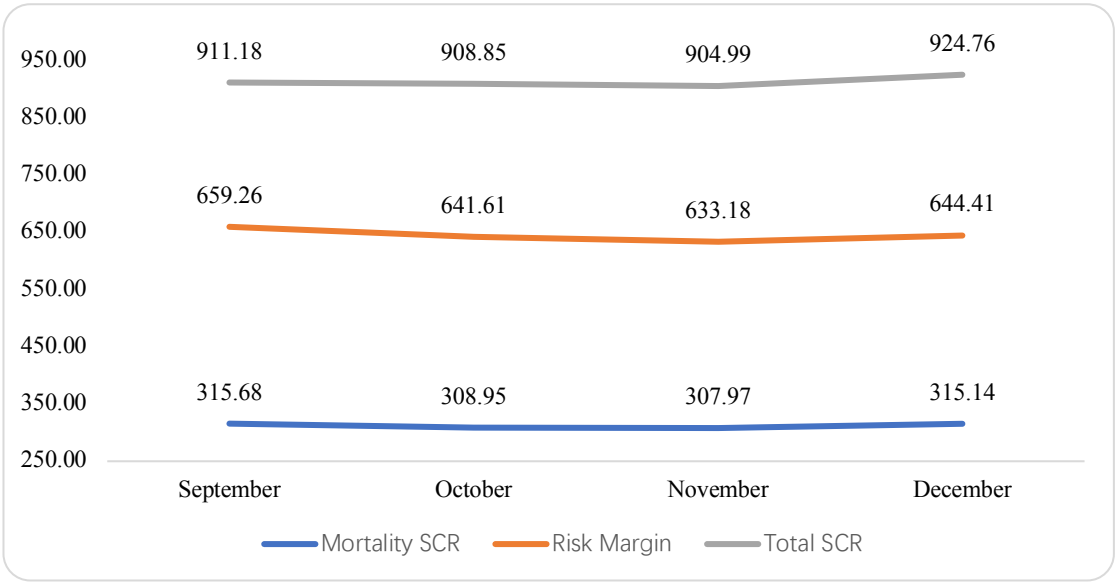
The other market risk is equity risk. Since we invest in stocks, the stock price will change over time, which will influence the Solvency II ratio. This one is easier than interest rate. Equity will affect assets and equity SCR. When equity increases, the assets will increase, and equity SCR will also increase. Since the Solvency II ratio is Own Funds (asset minus liability minus risk margin), and then divided by total SCR, there is not yet a clear relationship between equity and solvency II ratio.

After looking at the relationship between interest rates, equity and Solvency II ratio, we need to look at our company specifically. We need to analyze why October has the largest Solvency II ratio, and December has the lowest one. The following graph shows the different interest rates curves.

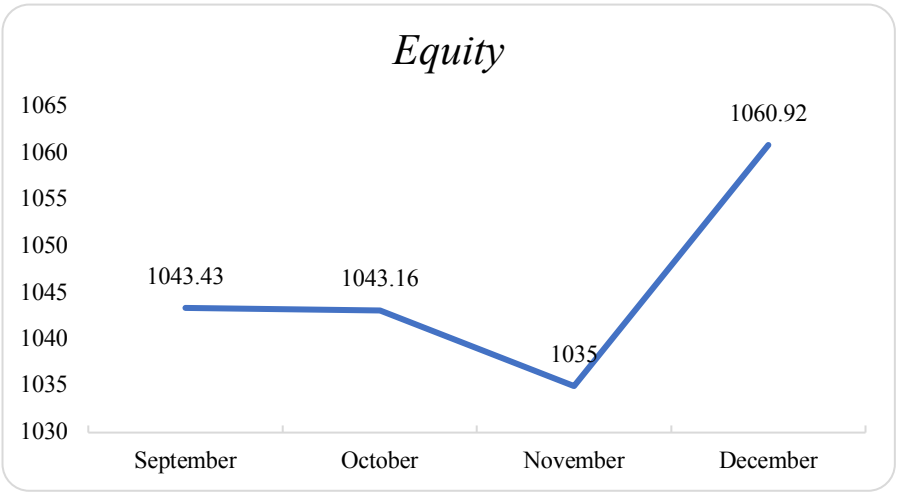


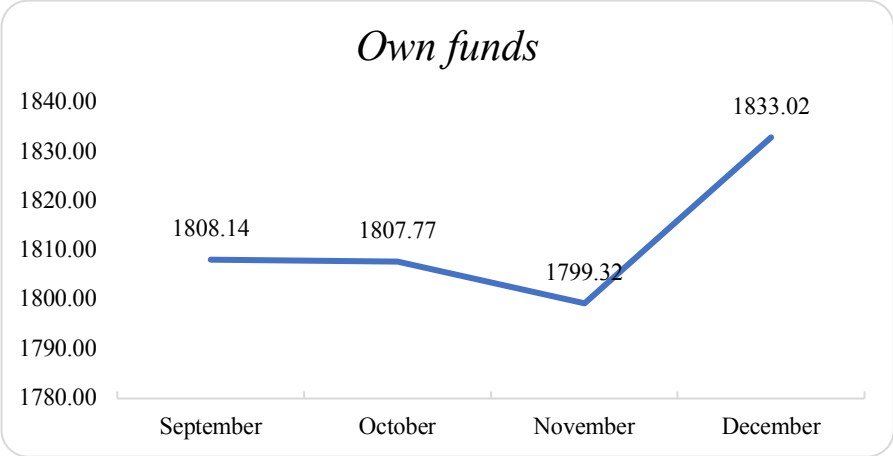
From the graph, we can see that the interest rate in November and October are relatively high, and December and September are relatively low. To demonstrate the explanation stated in the previous paragraph, we plot Mortality SCR, Risk Margin and total SCR. As stated, in October

and November, where interest rates are higher, mortality SCR, risk margin, and then total SCR are relatively low. To the contrary, in September and December, where interest rates are lower, mortality SCR, risk margin, and then total SCR are relatively higher, which demonstrate our deduction before.



Now we have showed that total SCR is relatively low in October. The lowest total SCR is in November. To show the reason why October has the lowest solvency II ratio, we need to consider the effect of equity. The following two graphs show equity and Own Funds (total asset minus liability minus risk margin).





According to this graph, we can see that equity in October is larger than equity in November. Therefore, Own Funds in October is also larger than equity in November. This explains why October has the largest solvency II ratio. This graph also explains why December has the lowest solvency II ratio. Notice that the largest amount of equity is in December. Although it has the largest own funds, it also has the largest total SCR. Since the total SCR in December is too large to offset, it has the lowest Solvency II ratio.

3. CONCLUSION

Our company has the lowest Solvency II ratio in December, which is 198.22%, and the largest Solvency II ratio in October, which is 198.91%. The two factors that influence the Solvency II ratio are interest rates and equity. The main reason that the solvency II ratio is the highest in October is that interest rates are relatively high, and equity is relatively high in October. For the lowest Solvency II ratio we have that the interest rates are relatively low, and equity is the highest in December. Overall, the change in Solvency II ratio is significant sophisticated. Every case needs to be analyzed separately.

Although the Solvency II ratios are different from September to December, these four numbers are close to each other. All in all, every Solvency II ratio from September to December for our company is larger than 150%, showing that our company is in a good financial condition.

REFERENCE:

"Risk-Free Interest Rate Term Structures." EIOPA, EIOPA, eiopa.europa.eu/regulation-supervision/insurance/solvency-ii-technical-information/risk-free-interest-rate-term-structures.

Official Journal of the European Union. European Union, 17 Jan. 2015, eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=OJ:L:2015:012:FULL&from=EN.

Oliver, Michael. Solvency II - The Story To Date. Gibraltar FSC, 25 Nov. 2009, www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=4&cad=rja&uact=8&ved=0ahUKEwiiytKz4qfXAhVjzoMKHYIjDfEQFghDMAM&url=http%3A%2F%2Fwww.fsc.gi%2Fdownload%2Finsurance%2FsolvencyII%2FGibraltar%2520Solvency%2520II%2520-%2520The%2520Story%2520To%2520Date%2520251109.pdf&usg=AOvVaw3yL98P67XvitOnUgbCKnDR.

"Solvency II Timeline." InsuranceERM, InsuranceERM, 22 Dec. 2015, www.insuranceerm.com/guides/solvency-ii-timeline.html.

Zaremba, Arthur J. "How to Estimate Risk Margins under Solvency II." PWC, PWC, 22 May 2012, www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwjApJTzz6jXAhVo0YMKHayCAR8QFggnMAA&url=https%3A%2F%2Fcas.confex.com%2Fcas%2Fspring12%2Fwebprogram%2FHandout%2FSession4857%2FSolvency%2520II%2520Risk%2520Margins%2520%252805-22-12%2529_CAS%2520Version.pdf&usg=AOvVaw2EDtP5gFvZJT1LmqIJU4k7.