

# Math 595: Tube Formulas in Convex and Riemannian Geometry

CRN 48012

Fall 2017

Part B (Oct 23 – Dec 13, 2017)

TueThu 9:30-10:50 am

445 Altgeld Hall

**Instructor:** Jeremy Tyson

**Office Location:** 329 Altgeld Hall

[tyson@illinois.edu](mailto:tyson@illinois.edu)

**Course Description:** Weyl's tube formula computes the volume of a small tubular neighborhood of a smooth submanifold of Euclidean space. Remarkably, the volume of the scale  $r$  neighborhood is a polynomial in  $r$ , whose coefficients encode various curvatures of the submanifold. There are similar formulas in convex geometry for the volumes of neighborhoods of convex sets, or more generally, Minkowski sums of convex sets. These sometimes go under the name *quermassintegrals* or *mixed volumes*, and go back to the work of Jakob Steiner. In this course we will discuss various tube formulas for neighborhoods of convex sets and neighborhoods of smooth submanifolds in Riemannian manifolds. At the end of the course, time permitting, we will indicate some recent extensions of these ideas to sub-Riemannian manifolds, specifically, the sub-Riemannian Heisenberg group.

**Textbook:** There is no required textbook. In the first part of the course I will roughly follow the book

*Tubes*, by A. Gray (2<sup>nd</sup> edition), Progress in Mathematics, vol. 221, Birkhauser, 2004

**Prerequisites:** A working knowledge of real analysis and differential geometry (at the level of Math 540 and Math 518) will be assumed. Familiarity with Riemannian geometry (as in Math 519) will be helpful, but we will review all necessary tools of Riemannian geometry as needed. In the final part of the course some prior exposure to the theory of Lie groups may be helpful.

**Grades:** Grading will be based on attendance and participation. Each student will be asked to give a short presentation on a topic related to the subject of the course during the final week of the semester.