MATH 595 Nilpotence and Periodicity in Stable Homotopy Theory

Second half of Fall 2018
(October 22 - December 12)

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Course description: One of the basic and motivating objects of study in algebraic topology are the stable homotopy groups of spheres, which are the endomorphisms of the unit object in the stable homotopy category. They form a graded commutative ring, $\pi_* S$, of which only $\pi_0 S$ is non-torsion, while every other element is torsion and nilpotent. This is probably the most complicated ring you may ever encounter, as its nature is completely elusive.

There is a filtration, called the chromatic filtration, that in principle helps one understand something structural about $\pi_* S$, and which led Ravenel to make an amazing list of conjectures about the structure of the stable homotopy category in [1]. Even more amazing were the proofs that followed shortly thereafter, by Devinatz, Hopkins, and Smith [2, 3]. These results brought Quillen's work on the relationship between formal group laws and homotopy theory to a whole new level, and have shaped homotopy theory and a number of related areas ever since.

In this course, we will start with a quick introduction of the players involved, largely following the first few chapters of Ravenel's book [4]. Then we will cover some of the work in [2,3] in detail. We will try to sketch the proof of the nilpotence theorem in as much detail as time allows; some input results may have to be black-boxed. Assuming the nilpotence theorem, we will prove the thick subcategory theorem.

Prerequisites: Familiarity with the basics of algebraic topology (eg. MATH 525 and 526), and especially homotopy theory (eg. MATH 527).

Texts include:

[1] Ravenel, Douglas C. *Localization with respect to certain periodic homology theories.* Amer. J. Math. 106 (1984), no. 2, 351-414.

[2] Devinatz, Ethan S.; Hopkins, Michael J.; Smith, Jeffrey H. *Nilpotence and stable homotopy theory. I.* Ann. of Math. (2) 128 (1988), no. 2, 207-241.

[3] Hopkins, Michael J.; Smith, Jeffrey H. *Nilpotence and stable homotopy theory. II.* Ann. of Math. (2) 148 (1998), no. 1, 1-49.

[4] Ravenel, Douglas C. *Nilpotence and periodicity in stable homotopy theory.* Ann. of Math. Stud., 128, Princeton Univ. Press, Princeton, NJ, 1992.