MATH 595 Advanced Topics in Mathematics: Water Waves Spring 2018

Water waves encompasses a wide range of phenomena, from ripples driven by surface tension to rogue waves and tsunamis. The phrase describes the situation, where water lies below a body of air and is acted upon by gravity.

While water waves have stimulated a considerable part of historical developments in the theory of wave motion, they present profound and subtle difficulties for rigorous analysis, modeling, and numerical simulation. Notably, the interface between the water and the air is a priori unknown and to be determined as part of the solution. Free boundaries are mathematically challenging in their own right. In addition, boundary conditions at the free surface are severely nonlinear, presenting further challenges.

We shall focus on some latest developments in the mathematical aspects of water waves. They include:

(1) Global regularity versus finite-time singularities for the Cauchy problem.
(2) Existence of traveling waves and their classification.
(3) Stability and instability of traveling waves.

Emphasis is on large scale dynamics and genuinely nonlinear behaviors, such as breaking and peaking, which ultimately rely on analytical proofs for an acute understanding.

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Text No official textbook. I will provide lecture notes and reading material.

Prerequisite MATH 540, MATH 553 or MATH 554 are useful, but not strictly required, provided that you have a strong undergraduate background on real analysis and PDEs and you are willing to work hard.