MATH 595—DEFINABILITY THEORY IN EXPANSIONS OF THE REAL LINE

Instructor: Philipp Hieronymi

Time: MWF 11-11:50AM
Place: TBA

Content. The goal of this topics course is to give an up-to-date overview of the state of the study of definable sets in expansions of the real line. In [1] Miller asked what became the overarching question of this research area: "What might it mean for a first-order expansion of the real line to be tame or well behaved?" Model theorists, real-analytic geometers and more recently number-theorists(!) have focused on the o-minimal setting: an expansion of the real line is o-minimal if every definable set has finitely many connected components. However, there are now many documented examples of expansions of even the real ordered group that define sets with infinitely many connected components, but still can be considered as tame (e.g., the topological closure of every definable set has finitely many connected components). The analysis of such expansions nowadays uses a fascinating combination of model-theoretic, metric-geometric, descriptive set-theoretic and automata-theoretic methods.

In this topics course I plan to give an updated version of the influential, but a bit out-dated survey paper [1]. I will review all basic notions (even o-minimality!) and cover the most important theorems in the area. Throughout the course I will try to state as many open and approachable problems as possible, giving interested students a chance to enter this research area.

Prerequisites. Students should have taken Math 570. Knowledge of Math 571 is helpful, but I will design the course such that knowledge of Math 571 is not necessary.

References

[1] Chris Miller, *Tameness in expansions of the real field*, Logic Colloquium '01, Lect. Notes Log., vol. 20, Assoc. Symbol. Logic, Urbana, IL, 2005, pp. 281–316. MR 2143901