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Graduate Course Description
Fall 2017 CRN: 64464
Math 595: Anatomy of Integers and Random Permutations

Instructor: Kevin Ford

Time/place: MWF 1–1:50; Altgeld 347

Prerequisites: Basic analytic number theory will be very helpful (equivalent of the first half of Math 531; elementary prime number estimates and multiplicative functions). Some knowledge of basic probability will be helpful but not necessary.

Recommended Texts: *Divisors*, by R. R. Hall and G. Tenenbaum, Cambridge Tracts in Math. **90**, paperback edition, 2008. (highly recommended for purchase).

Course Description.
Integers factor uniquely into a product of primes, and permutations factor uniquely into a product of cycles. Basic questions one can ask about these structures are

- How many prime factors does a typical integer $n \in [1, x]$ have? What is the distribution of those prime factors?
- How many cycles does a typical permutation of S_n have? How are the lengths of the cycles distributed?

Perhaps surprisingly, there is a close connection between these two problems, both distributions governed by the same probabilistic law. In the first part of the course we will examine carefully, on many scales, the distribution of the prime factors of typical integers and cycle decompositions of typical permutations. We will emphasize the connections between these two structures, stressing probabilistic techniques and ideas. In the second part, we will apply this knowledge to answer questions which about the distribution of divisors of integers, fixed sets of permutations (a subset of $\{1, \ldots, n\}$ which is itself permuted by the permutation) and applications of these bounds. Some examples:

1. How likely is it that an integer has two divisors in a fixed dyadic interval $(y, 2y]$?
2. How likely is it that an integer has two divisors in *some* dyadic interval $(y, 2y]$?
3. How many *distinct entries* are there in and $N \times N$ multiplication table?
4. How likely is it that a random permutation $\pi \in S_n$ contains a fixed set of size equal to k (that is, contains cycles with lengths summing to k)?
5. Given r random permutations of S_n, chosen independently, are there infinitely many k so that every permutation fixes a set of size k? We show that when $r = 3$, the answer is yes with probability 1, and when $r = 4$ the answer is no with probability 1.

Problems 1,2,3 are classical problems of Erdős. Problem 4 generalizes the classical derangement problem ($k = 1$). Problem 5 has application to "invariable generation of S_n".

Grades. The course grade will depend on homework assignments, which will be given periodically.