MATH 595 Fall 2018, Tu/Th 11:00-12:20

ALGEBRAIC STRUCTURES FOR MACDONALD THEORY AND INTEGRABILITY PART A AND B

PHILIPPE DI FRANCESCO

This is a 2 part, semester long, graduate topics course. The idea is to review the various algebraic structures underlying the theory of Macdonald operators, whose common eigenfunctions are the celebrated Macdonald polynomials. Most of these structures also underly continuous and integrable systems, i.e. systems with a lot of symmetries, allowing for exact solutions. We will present applications to models of statistical physics, as well as combinatorial problems.

The course is open to graduate students in mathematics or physics. We will review all the basic mathematical or physical material, so no extra knowledge is required. The format of the course is mainly lectures and study of research or expository papers.

We will explore properties of the symmetric group and some families of symmetric orthogonal polynomials, the Hecke algebra and its Temperley-Lieb quotient, the Double Affine Hecke Algebra and its functional representation leading to knot invariants, and the interplay between these algebraic structures and quantum cluster algebras attached to Q-systems, and their consequences for the study of characters of graded tensor products.

Part A

1. Symmetric group

Symmetric group, characters and Linear group Symmetric functions Orthogonal symmetric polynomials: Schur, Hall-Littlewood, Jack, Macdonald Related combinatorial problems

2. Hecke algebra HA/Temperley-Lieb algebra TLA

Integrable lattice models and general solutions of Yang-Baxter equation Ideals of TLA Dixmier trace and scalar product Gram matrix The Meander problem

Part B

3. Double affine Hecke algebra DAHA (type A)

Generators and relations Functional representation Macdonald operators Non-symmetric Macdonald polynomials $SL_2(Z)$ action Refined knot invariants Bosonization/plethysms

4. Quantum toroidal algebra qTorA and elliptic hall algebra EHA

Currents and relations Macdonald currents and level 0 representation Elliptic Hall algebra isomorphism Commutation relations for Macdonald operators q-Whittaker limit: quantum determinants and alternating sign matrices

5. From DAHA to cluster algebra CA: Q-systems

q-whittaker limit, Toda difference equation Quantum Q-system and Macdonald difference operators Graded characters Other types