# Spectral Theory of Partial Differential Equations

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Math 595 section SPD, Spring 2017
CRN: 64838

**When and where:** MWF 10-11am, room 445 AH
**March 6 to May 3** (2nd half semester)
Begins early (March 6) because class will not meet in the week April 3-7

**Course Material:** Spectral methods permeate the theory of PDEs and mathematical physics. Solving linear PDEs by separation of variables leads to eigenvalues when the spectrum is discrete, and continuous spectrum when it is not. Spectral properties then determine the stability of steady states and traveling waves, frequencies of oscillation, and energy levels of quantum systems.

Part I - Discrete spectrum
Wave, diffusion, Schrödinger, beam/plate (4th order) and variational methods
Computable special cases (interval, rectangle, disk, ball, harmonic oscillator)
Boundary conditions: imposed and natural
Geometric bounds for low eigenvalues, Weyl asymptotic for high eigenvalues
Can you hear the shape of a drum? (inverse spectral problem)
Applications to reaction-diffusion stability and thin fluid film stability

Part II - Continuous spectrum
Computable special cases
Schrödinger with potential
Self-adjoint operators

**What will this course be like?**

Research is different from coursework. Research often starts with questions motivated by analogy, or by trying to generalize special cases. Often we find answers in a nonlinear fashion, slowly developing a coherent theory by linking up and extending our scraps of known information. We cannot predict what we will need to know in order to succeed, and we certainly do not have enough time to study all relevant background material.

To succeed in research, we must develop a rough mental map of the surrounding mathematical landscape, so that we know the key concepts and canonical examples (without necessarily knowing the proofs). Then when we must learn more about a topic, we know where to begin.

This course aims to develop your mental map of spectral theory, as it relates to partial differential equations.

**Is this course right for you?**

If you have taken a graduate course in partial differential equations (e.g. Math 553), then give this course a try! Graduate students from other departments are welcome.

**Assessment**: based on a 20-30 minute presentation on a topic relevant to the course.