

Math 595: Advanced Topics in Mathematics
Title: Perron-Frobenius Operators (CRN 64835, Section PFO)
Spring 2017, Part A (Jan 17 - March 10)

Instructor: Florin P. Boca
Office: 371 Altgeld Hall
E-mail: fboca@illinois.edu

Course description: Perron-Frobenius operators provide a very powerful analytic tool in the study of the ergodic properties of piecewise continuous transformations $T : I \rightarrow I$, where $I = [a, b]$. For such a nonsingular transformation (i.e., $\lambda(A) \Rightarrow \lambda(T^{-1}A) = 0$ for every Borel set $A \subset I$, where λ is the Lebesgue measure on I), the corresponding Perron-Frobenius operator $P_T : L^1 \rightarrow L^1$ can be defined by the Radon-Nykodim derivative of the measure $T_*\lambda$ with respect to λ , as follows:

$$\int_A P_T f d\lambda = \int_{T^{-1}A} f d\lambda, \quad \forall f \in L^1, \forall A \text{ Borel set.}$$

The spectral properties of this operator, or of related operators associated with T and defined on other spaces of functions, provide a wealth of information about the ergodic properties of the original transformation, such as existence of absolutely continuous invariant measures, ergodicity, mixing, or distribution of periodic points.

This one-semester course will provide an introduction to this topic, focusing mainly on applications to transformations of number theoretical nature arising from continued fractions, such as Gauss and Farey type transformations.

Prerequisite: Math 540 and Math 541, or approval of instructor.
Textbook: There is no required textbook.