

MATH 595
THEORY OF PARTITIONS
11 A.M., 141 ALTGELD ON MWF

BRUCE BERNDT

Prerequisite: There is no formal prerequisite. However, Math 453, or an equivalent course in elementary number theory, and (possibly) Math 531 might be helpful.

This is an introductory course in the theory of partitions. No prior knowledge of the subject is necessary. However, I will seriously endeavor to make the course interesting to students with a variety of backgrounds.

The partition function $p(n)$ is defined to be the number of ways the positive integer n can be expressed as a sum of positive integers, with the order of the summands irrelevant. One can define many other partition functions, where restrictions are put on the summands. For example, one may add restrictions on the size of the parts, the number of parts, or the residue classes in which the parts lie. Elementary, combinatorial, graphical, algebraic, and analytic methods can be used to study partitions. Our primary approach in the study of partitions will be through q -series. However, there will be a strong emphasis on elementary and combinatorial methods as well. The necessary material from q -series will be developed *ab initio*. Much of the material on q -series that we will need can be found in G. E. Andrews' book, *The Theory of Partitions* or in the lecturer's monograph, *Number Theory in the Spirit of Ramanujan*.

Topics included in the course are:

- (1) Ramanujan congruences for $p(n)$
- (2) the parity of $p(n)$
- (3) the famous Rogers–Ramanujan identities
- (4) Schur's identities and the Ramanujan–Göllnitz–Gordon identities
- (5) the smallest parts partition function
- (6) overlined partitions
- (7) partition functions connected with mock theta functions
- (8) plane partitions
- (9) Frobenius partitions

The lecturer has typed course notes from the last time that he taught the course. Further typed notes will be prepared. Some of our beginning lectures will be taken from the two sources mentioned above.