Mathematics 595 — Higher category theory and quasicategories

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Course description:
Higher category theory is the study of structures which are like categories, but are "higher-dimensional": while a category has objects (0 dimensions), and morphisms between objects (1 dimensions), higher dimensional analogues are allowed to have morphisms between morphisms (2 dimensions), and so on.

The goal of this course is to descibe an approach to this called *quasicategories*. These were invented by Boardman and Vogt, and were developed further by André Joyal (in various papers and unpublished preprints) and Jacob Lurie (in his book *Higher topos theory*, where he calls them ∞-categories).

The goal of this course is to give an *accessible* introduction to this theory. That is, we imagine that we are familiar with classical category theory, and we are confronted with the strange new notion of a quasicategory. We will attempt to develop basic concepts and results for quasicategories by analogy with what we know about classical categories.

This is, of course, not necessarily straightforward. In order to proceed, we will need to develop "simplicial homotopy theory" and the theory of "model categories". These are not prerequisites: they will be introduced and developed through the course.

Prerequisites: Some familiarity with the basic notions of classical category theory is necessary (e.g., functors, natural transformations, limits and colimits, etc.)

Familiarity with basic algebraic topology (e.g., fundamental group and singular homology, as in Math 525), or homological algebra, will be helpful, but not essential.

Texts: I will try to produce lecture notes concurrently with the course. Important references for the material of this course include

- Joyal, "Quasi-categories and Kan complexes", J. Pure Appl. Algebra 175 (2002).
- Joyal, "Notes on quasicategories", preprint (2008).
- Joyal, "The theory of quasi-categories and its applications", preprint (2008).
- Lurie, *Higher Topos Theory*, (2009).
- Riehl and Verity, "The 2-category theory of quasicategories", Adv. Math. 280 (2015).