

Math 595

Two Theorems in Contact and Symplectic Topology

Instructor: Ely Kerman

Lectures: Tuesday-Thursday 11:00 to 12:20

Course Description

In this class we will introduce the fields of Contact Topology and Symplectic Topology, from the top down, by taking a long hard look at two of the landmark theorems in these areas.

We will first consider Yasha Eliashberg's beautiful classification of contact structures on three manifolds. Along the way we will encounter many related topics including homotopy theory, surgery theory, the h-principle and dynamics in low dimensions. This area of contact topology is currently undergoing a period of dramatic growth and much of this progress is based on the foundational ideas that underlie Eliashberg's classification theorem.

Our second big theorem will be Andreas Floer's proof of Arnold's Conjecture for Hamiltonian diffeomorphisms. These diffeomorphisms include the flows of classical mechanical systems. They preserve volumes and, more mysteriously and profoundly, a certain 2-form called a symplectic form. In a paper which essentially started the field of symplectic topology, Arnold famously conjectured that such maps should have more fixed points than are predicted by standard tools from differential topology such as the Lefschetz Fixed Point Theorem. Amazingly Floer's proof of this conjecture turned out to be even more influential than the conjecture itself. It resulted in the construction of what is now called Floer homology a tool which is still being intensely developed and used in many fields of mathematics. In understanding Floer's proof we will learn the basic construction of Lagrangian Floer homology.

Prerequisites

Basic differential topology at the level of Math 518 is recommended.