# Math 595: Local fields
# Fall 2016

**Instructor:** Patrick Allen (pballen@illinois.edu)

**Description:** In the late nineteenth century Kurt Hensel introduced the field of $p$-adic numbers $\mathbb{Q}_p$, which can be thought of as the completion of $\mathbb{Q}$ with respect to a metric that keeps track of arithmetic tied to the prime $p$, unlike the real numbers. His motivation was to use power series methods to study problems in number theory, as the field $\mathbb{Q}_p$ behaves in many ways similar to the field of Laurent series $\mathbb{F}_p((T))$ over the field of $p$-elements $\mathbb{F}_p$. In fact, both are examples of *non-archimedean local fields*, and their study has applications to understanding number fields and function fields of curves defined over finite fields.

In this course we will develop the theory of non-archimedean local fields in detail. We will begin by describing their internal structure and arithmetic. We will then discuss finite extensions of local fields, their ramification, and their Galois groups. We will then move to one of the main topics of the course: local class field theory.

Local class field theory shows, in a precise and canonical way, that the internal arithmetic of a local field determines the abelian Galois extensions of the field. The main theorems of local class field theory are not only very elegant, but also very useful. The proofs are involved, but will allow us to develop some tools that are interesting in their own right, and are useful in other areas of mathematics.

Time permitting, we will then initiate a study of the representations of the absolute Galois group of a local field. We will only be able to scratch the surface, but we'll provide some vistas of where the theory goes from here.

**Prerequisites:** Math 500 (Graduate Algebra) or equivalent.

**Grading:** Grades will be determined based on assigned homework and presentations.

**References:** There is no required text. The following references may be useful:

Iwasawa, *Local class field theory*
Serre, *Local fields*