OPEN PROBLEMS IN GROUP THEORY AND TOPOLOGY

Igor Mineyev. Math 595, second half of Fall 2018 (October 22 - December 12), MWF 3pm.
`www.math.uiuc.edu/~mineyev/class/18f/595/`

There is a deep relationship between groups, topological spaces, and metric spaces. Studying one helps studying another. Groups can be viewed as geometric objects, to any topological space one can associate its fundamental group, group presentations lead to cell complexes, metric spaces can be studied using group actions, etc. Geometric group theory is the area of mathematics investigating such relations.

In this course I will concentrate on multiple (and very difficult) open problems that emphasize such relations. If you can solve *any one* of those, turn it to me as homework. Here are open-ended themes that I plan to address in the course.

(1) 1-complexes: Cayley graphs of groups, free groups and their subgorups, Nielsen transformations, the Hanna Neumann conjecture (solved), generalizations of HNC, ...

(2) Free groups and free products: one-relator groups, Magnus' *Freiheitssatz* and its generalizations, the Kervaire conjecture, ...

(3) 2-complexes: group presentations, (highly nontrivial) presentations of the trivial group, the Andrews-Curtis conjecture, the Grigorchuk-Kurchanov conjecture, aspherical 2-complexes, the Whitehead conjecture, the Bestvina-Brady construction, ...

(4) Geometric methods: hyperbolic groups and spaces, the boundary at infinity of a hyperbolic group, Cannon's conjecture, ...

(5) 3-manifolds: the Poincaré conjecture (from a group-theoretic perspective), Stallings' approach ("How not to prove the Poincaré conjecture"), surfaces in 3-manifods, ...

If you are interested in exploring these realms deeper, see the references to books on this class website. This course will only run if sufficiently many students sign up.