

# Math 595: Factorization algebras and factorization spaces

Fall 2018

**Instructor:** Emily Cliff.

**Course content:** Beilinson and Drinfeld introduced the notion of factorization algebras and factorization spaces, geometric incarnations of the notion of a vertex algebra. Vertex algebras have an important role to play in representation theory, especially the representation theory of infinite-dimensional Lie algebras. They are also important in mathematical physics, since they describe the symmetries of two-dimensional conformal field theories.

The study of factorization structures allows us to approach these questions using insights and tools from algebraic geometry. Another advantage of this perspective is that the definitions extend naturally to higher dimensions (as shown by Francis and Gaitsgory), whereas the definition of a higher-dimensional analogue of a vertex algebra is still unknown.

In this course, we will begin by extending our foundations of algebraic geometry to cover the necessary classes of spaces and the (differential-graded) categories of sheaves on these spaces that we will need. Next, we will introduce the notions of factorization spaces, factorization algebras, and chiral algebras, and study the relationships between them.

We can then study properties of factorization algebras/chiral algebras, including commutativity, universal chiral envelopes, and factorization/chiral homology. We will investigate fundamental examples (for example, the Beilinson–Drinfeld affine Grassmannian, and the factorization algebras associated to it) to see how they fit into different areas of mathematics. There will be some flexibility in the course to address topics of particular interest to the participants.

**Prerequisites:** Familiarity with the language of schemes and sheaves will be assumed, as will a basic knowledge of category theory (adjunctions, limits, colimits, etc.). Knowledge of  $D$ -modules will be helpful, but is not required. Prior familiarity with vertex algebras or with mathematical physics is not at all expected.

Students who are interested in the material but not sure whether they have enough background knowledge are welcome to contact me to discuss the matter.