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Message from the Director

The Illinois Geometry Lab continues to evolve in new and exciting ways. Our research efforts are as strong as ever, with 32 students participating in 11 projects in Spring 2014. We’re now involving high school students from across the country in an innovative partnership with PRIMES, an MIT-based organization for high school research in mathematics. Our researchers have presented at national and international conferences, including our own Illinois Undergraduate Research Symposium and Engineering Open House, and have won several poster and presentation prizes.

Our public engagement efforts have grown exponentially, thanks to the efforts of our Associate Director of Public Engagement, Dr. Noel DeJarnette. Partnerships with over a dozen organizations at local, national, and international levels have enabled us to bring our message of “math is play” to many different audiences. To mention a few: we hosted a booth at the American Association for the Advancement of Science (AAAS) Family Science Days, with over 1000 visitors, in February, and are currently working with the ChiPrep college preparatory academy on a 4-week curriculum on the mathematical theory of polyhedra and its applications. We’ve also continued our collaboration with the Chirag School in the Himalayan mountains of northern India.

Finally, none of this would be possible without extensive support from the Department of Mathematics at the University of Illinois at Urbana-Champaign, the Office of Public Engagement, the Provost’s Initiative on Teaching Advancement, the Office of Undergraduate Research, and the National Science Foundation.

—Jayadev Athreya
Director, Illinois Geometry Lab
Lithium-Ion Batteries
Faculty Mentors: Jayadev Athreya, Yuliy Baryshnikov, Anil Hirani
Team Leader: Kaushik Kalyanaraman
Scholars: Justin Faber, Cheng Wang, Jialiang Wang
The goal of our project is to design the ideal model of a Lithium-ion battery with the most efficient charging time. We analyzed two different geometric properties of configurations: tortuosity and degrees of freedom of the particles. Based on the simulation of the charging process of batteries, we are trying to optimize the score function related to the whole system. The hope is that the score function can replace the simulation from last year and find optimal configurations of particles. We also used Python to simulate a Poisson model of the battery charging for different configurations with various positions, sizes and shapes of the particles using the finite element method (FEM). Further, we will quantify the various configurations using an appropriate objective function.

Visibility in Random Forests
Faculty Mentors: Jayadev Athreya, Francesco Cellarosi
Team Leader: Amita Malik
Scholars: Abigail Turner, Ananya Uppal, Peng Xu
If you stand at the origin and there is a tree at each integer point in the plane, then only a fraction of the trees in this infinite forest will be visible to you. Surprisingly enough, this fraction is $\frac{\zeta}{2}$. Last semester, we examined the gap distribution between these visible trees and how this distribution changes when you remove a randomly picked set of trees from the either the whole forest or the primitive forest. This semester we built on this by focusing on the distribution formed by the convolution of multiple Halls distributions. Although Halls distribution has infinite variance, we showed both numerically and by using a variation of the Central Limit Theorem that this distribution also goes to the normal distribution. We have been looking at the relationship between the distribution formed by removing randomly chosen subsets of trees from the primitive forest and the convolution of multiple Halls distributions.
Stability of Quasicrystalline Frameworks

Faculty Mentor: George Francis
Team Leader: Eliana Duarte

Scholars: Zach Miksis, Daniel Pugliese, Joseph Zeller

Let F be a deformable quasicrystal framework in 2D or 3D. Any face of F can be made rigid by bracing it with a rhombic plate. Bracing enough faces will make the framework rigid. How many faces, and in what configuration, should we brace to make F rigid? In the 2D case the solution is obtained by associating a graph to the bracing of the framework F. For this case we present the Wester Game which allows the user to explore the deformations of the framework in the plane. To study the 3D case we generalize the ideas we used to create the Wester Game and we present a real-time interactive animation to study coherent deformations of a cubical framework. For such a framework, we define a tunnel to be a maximal succession of adjacent cubes in the same direction. We conjecture that any 3D framework can be made rigid by plating one face in every tunnel of the framework.

Symmetry, Fractals, and Chaos in Number Theory

Mentor: A.J. Hildebrand
Team Leader: M. Tip Phaovibul

Scholars: Yiwang Chen, Daniel Hirsbrunner, Tong Zhang

Many properties of the natural numbers can be encoded as sequences of +1’s and -1’s. On the surface, such sequences often show no obvious pattern and indeed seem to behave much like randomly generated sequences. In this project we explore these sequences geometrically through certain "random walks" in the plane. The graphs of these random walks reveal a surprising variety of features: some have unexpected symmetries, others show fractal-like patterns, and yet others exhibit a more chaotic behavior. Depicted below are three examples of such random walks, constructed with the Moebius function (left figure), the sum-of-digits function (right figure), and with random coefficients (bottom figure).
Percolation Theory
Faculty Mentor: Kay Kirkpatrick
Team Leader: Tom Mahoney
Scholars: Yibo Guo, Yuxi He, Kaiyue Hou
In a network where nodes are potential hosts and edges represent contact between hosts, bootstrap percolation is used to study the spread of an infection throughout the graph. Each node becomes infected if enough of its neighbors are infected. Subject to the infection rule for each vertex and some initial seed of infected nodes, we ask if the infection spreads to the whole network. The infection continues to spread iteratively until no new nodes can become infected.

Given a network and set of infection thresholds for each node, the goal is to minimize the size of the initial seed needed to infect the whole network. Our results focus on studying regular lattices with constant infection thresholds for each node and with random infection thresholds for each node. We studied the terminal states on square and hexagonal lattices, as well as the minimum initial seed and how initial seed probability affects steps to terminal state. Since the question for square lattice is mostly solved, we focused more on hexagonal lattice.

Tiling Harmonic Functions
Faculty Mentor: Sergiy Merenkov
Team Leader: Vyron Vellis
Scholars: Yilun Du, Qing Ma, Sufe Zhang
Suppose that $T$ is a square tiling in the complex plane, i.e. a finite collection of mutually disjoint squares with edges parallel to $x$ and $y$ axes. If $u$ is a function defined on the vertices of $T$ then the energy of $u$ on $T$ is

$$E_T(u) = \sum_{t \in T} (\max_{v \in t} u(v) - \min_{v \in t} u(v))^2$$

where sum is taken over all squares $t$ of $T$ and max, min are taken over vertices $v$ of $t$. Given a tiling $T$ and values on the boundary vertices of $T$, which function defined on all vertices of $T$ minimizes the energy? In this project we have developed a computer algorithm which calculates energy minimizers. Such minimizers are called tiling-harmonic functions. Furthermore, we compare this concept of harmonicity with the well known concept of graph-harmonic functions.
Symmetries in Nature
Faculty Mentor: Laura Schaposnik
Team Leader: Junxian Li

Scholars: Chaofan Da, Chris Formos, Michelle Zosky
The goal of this project is to study symmetries appearing in nature, particularly in the formation of snowflakes and viruses. This project revolves around three main themes:

- (I) The study of the basic geometric properties of snowflakes and viruses, understanding their differences and similarities.
- (II) The mathematical modeling that has been developed to visualize both snowflakes and viruses.
- (III) The production of 3D printed samples and a school curricula that can be used to teach the basic mathematical concepts involved to elementary and primary school students.

Having studied the basic mathematics involved in the geometry of regular snowflakes and icosahedral viruses, we are now focusing our attention on (II) and (III). In particular, we have been able to follow past work by Reitler and reproduce the snowflake growth algorithm, which allows us to generate images as bellow. Finally, we have begun the study of further questions, which will continue next semester:

- (IV) Icosahedral viruses have an invariant, its Triangulation number, and only for some of its values the viruses have been described in the past. We are now looking at mathematical reasons why viruses for other values would not exist, and in collaboration with some biologists, we are studying more subtle geometrical properties of viruses.
- (V) The existing algorithms, which describe snowflake growth, do not take into account the local geometry. We are thus trying to improve these algorithms to include, for example the diffusion effect.

Interacting Particles
Faculty Mentor: Ken Stolarsky
Team Leader: Mike DiPasquale

Scholars: Konrad Wrobel, Hang Yang, Mingxue Zhu
If 5 electrons are placed on a metal sphere, where will they end up when they stop the motion caused by their mutual repulsion? If 5 points are placed on a sphere so that the smallest distance between any two is maximal, what are the possibilities? What if the sum of all ten distances between them is maximized? Must the center of mass be the center of the sphere? The above are problems about interacting particles. Even in the case where all particles are identical there is surprisingly little that has been proved. Some statements about electrons by Maxwell himself still constitute unsolved problems, as does the (no longer used by physicists) plum pudding model of the atom by J.J. Thomson. In this project we investigate interacting particles in two dimensions using Mathematica to visualize the resulting configurations. Symmetries abound!
Geometric Model of the Visual Cortex

Faculty Mentor: Jeremy Tyson
Team Leader: Anton Lukyanenko
Scholars: Jon Graven, Nishad Phadke, Mohammad Saad

Our goal is to model how the brain begins to fill in missing information in visual information. Given an occluded, or incomplete, image, we use a series of 12 CORF filters to mimic the responses of simple cells in the visual cortex. Each filter is tuned to a different preferred angle, just like the neurons they are modeling. These response values are inserted into a discrete graph that limits connections between cells to specific neighbors. The connections allow two different motions: movement along the preferred angle of the neuron and a slight change of preferred angle. We diffuse the response values using these connections in order to complete contours across occlusions in the image and fill in the missing data.

Community Involvement

Spring 2014 was an exciting semester of firsts for the Illinois Geometry Lab. Scholars from the lab were in high demand as we increased from one event per month to over one event per week!

This semester saw the design and implementation of new lessons involving 3D modeling, Calculator-Based Rangers™, Oculus Rift head mounted displays, see-saws, and conduit benders. IGL scholars showed their work and our new toys at four local Science Nights. We also participated in QED Math Symposium, AAAS Family Science Days, LEGO Day at the Orpheum Children’s Science Museum, Agora Days at University High School, and Engineering Open House—all for the first time.

This summer will mark our first involvement with local summer camps. We also had a number of new lab and classroom visits with students from after-school groups like the Girls Do Science Club, Cub Scout Pack 98, and the Douglass Community Center.

While we have added new partners, we are grateful for the continued interest from our longtime partners at Countryside Middle School, Illinois MakerLab, Urbana Middle School, Education Justice Project, Science at the Market, and the University Primary School.

IGL is proud to have led over two dozen events this semester, with over a thousand participants enjoying hands-on activities.

Figure 1: (a) The original image (b) The image formed purely from modeled neuron responses (c) Mid-way through diffusion (d) Modeled responses after diffusion.
About the Illinois Geometry Lab

The Illinois Geometry Lab is a facility at the Department of Mathematics at the University of Illinois focusing on mathematical visualization and community engagement.

At the lab, undergraduate students work closely with graduate students and postdocs on visualization projects set forth by faculty members. In the community engagement component of the lab, IGL members bring mathematics to the community through school visits and other activities.

To schedule an IGL activity with your group, contact the IGL outreach manager at igl-outreach@math.uiuc.edu.

Joining the Illinois Geometry Lab

We invite all undergraduate students to apply. Some familiarity with programming and completion of multivariable calculus are recommended, but prerequisites vary by project.

For additional information on joining the lab, visit:
http://www.math.illinois.edu/igl/join.htm

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