Mathematics 500 — Abstract Algebra I

Fall 2022
(11–12 MWF, in 347 Altgeld)

Instructor: Charles Rezk
Office: 257 CAB
Email: rezk@illinois.edu
Webpage: `http://faculty.math.illinois.edu/~rezk/`

Homework: Weekly homework assignments. (50% of grade.)

Tests: Two midterms (10% each) and a final (30%), in class. The final exam will resemble a Comp Exam.

Texts: The primary text will be:

- Dummit & Foote, *Abstract Algebra,* (3rd edition). Wiley, ISBN 978-0-471-43334-7.

Course topics: The course will cover approximately chapters 1–8, and 10–14 of Dummit and Foote. It will be assumed that students are familiar with basic material from an undergraduate algebra class, such as in Math 417 (this material will be reviewed). We will cover the topics on the standard syllabus for 500, including:

- Free groups and presentations of groups.
- Group actions.
- The Sylow theorems.
- Basic ring theory.
- Basic module theory.
- Classification of modules over a PID.
- Fields and field extensions.
- Galois theory.

INTRODUCTION TO GEOMETRIC GROUP THEORY

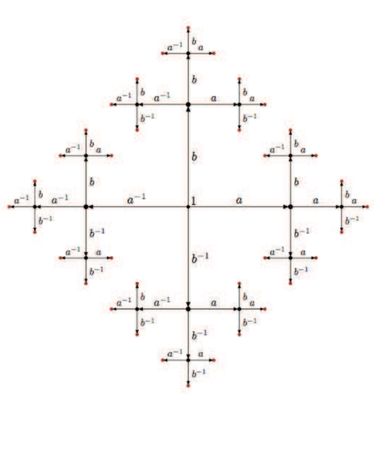
Igor Mineyev. Math 503, Fall 2022.
https://faculty.math.illinois.edu/~mineyev/class/22f/503/

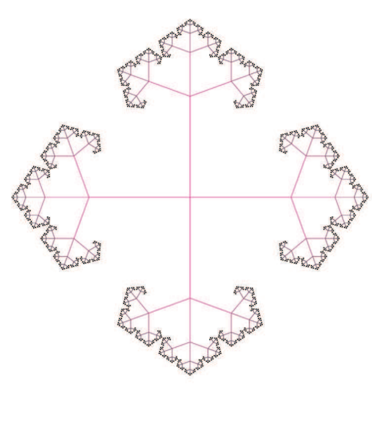
Geometric group theory is not a subject in itself; it is rather the place where various areas of mathematics interact: algebra, topology, geometry, analysis, computational methods, and more. Here is the tentative list of topics that I intend to cover in this course; this might be modified somewhat as we proceed.

- Cayley graphs, the word metric, groups as metric spaces, quasiisometry.
- One-dimensional things: Free groups and their subgroups, their descriptions via Stallings' graphs, Schreier's subgroup theorem, Nielsen transformations, automorphisms of free groups.
- Group actions on trees, free products, ping-pong lemma, free products with amalgamations, HNN-extensions, graphs of groups.
- Two-dimensional things: Groups presentations by generators and relators, van Kampen diagrams, van Kampen theorem, isoperimetric function, algorithmic problems in group theory.
- Examples of quasiisometry invariants: growth of finitely generated groups, ends, isoperimetric functions, amenability, solvability of the word problem, asymptotic cones, hyperbolicity.
- Multi-dimensional things: Word hyperbolic groups and spaces, their numerous definitions and properties, examples, the ideal boundary, quasiconformal and conformal structures on the ideal boundary, cubical complexes, ...

No textbook is required. The following sources might be helpful, among many other.

- Magnus, Karras, Solitar. Combinatorial group theory.
- Lyndon, Schupp. Combinatorial group theory.
- Jean-Pierre Serre. Trees.
- Ghys, Haefliger, Verjovsky. Group theory from a geometrical viewpoint.
- Collins, Grigorchuk, Kurchanov, Zieschang. Combinatorial group theory and applications to geometry.
- John Meier. Groups, graphs and trees.
- Gilbert Baumslag. Topics in combinatorial group theory.
- Bridson, Haefliger. Metric spaces of non-positive curvature.





FALL 2022
MATH 514
COMPLEX ALGEBRAIC GEOMETRY

Instructor: Sheldon Katz

Time: MWF 2–2:50

Text: C. Voisin, Hodge Theory and Complex Algebraic Geometry, I

Prerequisite: Math 448 or equivalent, or permission of the instructor

This course develops the theory of complex manifolds, with particular attention to compact Kähler manifolds, which include complex projective varieties. In the process, a foundation is established for further study in complex geometry and algebraic geometry. Geometric methods frequently come to the forefront in a complex analytic approach.

The course will include most of the first two chapters of the text, plus some related material from other sources. Selected topics from Chapters III and IV will be included as time permits.

Topics covered include complex manifolds, holomorphic vector bundles, Hermitian differential geometry, Hodge theory for Kähler manifolds, Dolbeault cohomology, and Hodge theory for Kähler manifolds. If time allows, applications will be given, including Hodge structures, the Lefschetz $(1,1)$ theorem, and the Kodaira embedding theorem.

A complex manifold of dimension 1 is a Riemann surface, so prior exposure to Math 510 will help, but is not a prerequisite. However, prior familiarity with complex analysis at least at the advanced undergraduate level is essential.

* 518 Differentiable Manifolds I

Differentiable manifolds are a class of spaces that includes Euclidean spaces, smooth curves and surfaces in 3-space, higher-dimensional generalizations such as the n-dimensional spheres, and infinitely much more. Manifolds locally look like open sets in Euclidean space, but they may have a nontrivial global topology. A differentiable manifold is one where, at least locally, we can "do multivariable calculus" in a meaningful way. This local structure has consequences for the global topology. Even for Euclidean spaces (whose global topology is trivial) the theory of manifolds provides a new perspective on geometry that vastly generalizes traditional Euclidean geometry in multiple directions.

The foundations of differentiable manifolds are not particularly simple, but they support a theory that has a lot of intuitive appeal. One goal of this course is to convey both the technical foundations and the intuitive picture.

The topics for this course are:

- Foundations of Differentiable Manifolds: Differentiable manifolds and differentiable maps. Tangent space and differential. Immersions and submersions. Embeddings and Whitney's Theorem. Foliations. Quotients.
- Lie Theory: Vector fields and flows. Lie derivatives and Lie brackets. Distributions and Frobenius' Theorem. Lie groups and Lie algebras. The Exponential map. Transformation groups.
- Differential Forms: Differential forms and Tensor fields. Differential and Cartan Calculus. Integration on manifolds and Stokes Formula.

The lectures will follow Prof. Rui Loja Fernandes' lecture notes on Differential Geometry. Other recommended textbooks are
- Introduction to Smooth Manifolds by John M. Lee.
- A Comprehensive Introduction to Differential Geometry by Michael Spivak.

Grades will be based on homework (25%), a midterm exam (25%), and a final exam (50%).

Mathematics 526 — Algebraic Topology II

Fall 2022
(1–2 MWF, in 447 Altgeld)

Instructor: Charles Rezk
Office: 257 CAB
Email: rezk@illinois.edu
Webpage: `http://faculty.math.illinois.edu/~rezk/`

Course description:
This is second semester course in algebraic topology. In the first semester (Math 525), invariants called homology groups were constructed in terms of the singular chain complex of a space. One of the themes of this course is thinking about the singular chain complex itself as as a kind of invariant, from which other invariants (homology and cohomology groups, possibly with coefficients) can be derived, as well as additional structure on them (cup products, cohomology operations).

Homework: There will be approximately six homework assignments, to be given out approximately once every two weeks.

Prerequisites: Math 525, or instructor consent.

Texts: The primary text will be:

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2001. This book is also available for free at
`http://www.math.cornell.edu/~hatcher/`

This will be supplemented with additional course notes. Other useful references include:

- Bredon, *Geometry and Topology.*
- Bott & Tu, *Differential Forms in Algebraic Topology.*
- Davis & Kirk, *Lecture notes in Algebraic Topology.*
- May, *A Consise Course in Algebraic Topology.*

Course topics:
The course will include the following topics.

- Singular cohomology.
- The cup product and the Künneth theorem.
- Cech cohomology and its relationship to singular cohomology.
- Poincaré Duality.

After this, some of the following topics may be covered, time and student interest permitting.

- Classifying spaces and characteristic classes.
- Spectral sequences and applications.
- Cohomology operations.
- Basic homotopy theory and homotopy groups.

MATH 531: ANALYTIC NUMBER THEORY, I
THE DISTRIBUTION OF PRIME NUMBERS

Kevin Ford (304 Altgeld, 265-6255, `http://www.math.uiuc.edu/~ford/`)

Prime number theory has witnessed many exciting new developments in the past few years:

- The primes contain arbitrarily long arithmetic progressions (Green and Tao, 2005)
- Bounded gaps containing many primes (Zhang, Maynard, Tao 2013–2014)
- New bounds for large gaps between primes (Ford, Green, Konyagin, Maynard, Tao 2014–16)
- Every odd number greater than 5 is the sum of three primes (Helfgott, 2014)

All of these rely on **analytic methods**, that is, methods stemming from some kind of analysis (broadly speaking, this included real analysis, complex analysis, and harmonic analysis).

Syllabus:

1. Arithmetic functions: theory of multiplicative and additive functions, Dirichlet convolution, Möbius inversion, average order of magnitude estimates. Big-O and little-o notation. Elementary theory of the distribution of primes, estimates of Chebyshev and Mertens.

2. Study of arithmetic functions via the analytic theory of Dirichlet series, Euler products and Perron's inversion formula.

3. Analytic methods for counting primes and prime in progressions. Theory of the Riemann Zeta function $\zeta(s)$ and Dirichlet L-functions $L(s, \chi)$.

4. Analytic proof of the Prime Number Theorem and the Prime Number Theorem for Arithmetic Progressions. Importance of the location of zeros of $\zeta(s)$ and $L(s, \chi)$. Discussion of the Riemann Hypothesis, Extended Riemann Hypothesis, and Landau-Siegel zeros.

5. The large sieve and the proof of the Bombieri-Vinogradov theorem. Consequences of the Elliott-Halberstam conjecture.

6. As time permits, a brief "sneak preview" of other further topics in analytic number theory, such as exponential sums, sieve methods, modular forms.

Text: Highly recommended for purchase:

H. Davenport, *Multiplicative number theory*, 3rd. ed, 2000.

G. Tenenbaum, *Introduction to analytic and probabilistic number theory, 3rd ed.*, 2015

Math 542 Complex Variables I, Fall 2022

M. Burak Erdoğan

This is a standard introductory course in complex analysis. Topics will include:

1. Complex number system. Basic definitions; topology of the complex plane; Riemann sphere, stereographic projection.
2. Differentiability. Basic properties; Cauchy-Riemann equations, analytic functions.
3. Elementary functions. Fundamental algebraic, analytic, and geometric properties. Basic conformal mappings.
4. Contour integration. The Cauchy integral theorem; consequences.
5. Sequences and series. Uniform convergence; power series.
6. The local theory. Zeros, Liouville's theorem, Maximum modulus theorem, Schwarz's Lemma.
7. Laurent series Classification of isolated singular points; Riemann's theorem, the Casorati-Weierstrass theorem.
8. Residue theory. The residue theorem, evaluation of improper integrals; argument principle, Rouche's theorem, the local mapping theorem.
9. The global theory. Winding number, general Cauchy theorem and integral formula; simply connected domains.
10. Uniform convergence on compacta. Ascoli-Arzela theorem, normal families, theorems of Montel and Hurwitz, the Riemann mapping theorem.
11. Infinite products. Weierstrass factorization theorem.
12. Runge's theorem. Applications.
13. Harmonic functions. Basic properties; Laplace's equation; analytic completion; the Dirichlet problem.

Prerequisites: MATH 446 and MATH 447, or MATH 448.

Textbook: An Introduction to Complex Function Theory, B. Palka.

Math 562 (Probability II)

Instructor: Renming Song
Office: 227 CAB
Phone number: 217 244 6604

Text: Jean-Francois Le Gall : Brownian Motion, Martingales and Stochastic Calculus, 2016, Springer

Course Topics: This is the second half of the basic graduate course in probability theory. This course will concentrate on stochastic calculus and its applications. In particular, we will cover, among other things, the following topics: Brownian motion, stochastic integrals, Ito's formula, martingale representation theorem, Girsanov's theorem, stochastic differential equations, connections to partial differential equations. If time allows, I will also present some applications to mathematical finance.

Math 561 is a prerequisite for this course. However, if you have not taken Math 561, but are willing to invest some extra time to pick up the necessary materials from 561, you may register for this course.

Grading Policy: Your grade will depend on homework assignment and a possible final exam.

Math 564: Applied Stochastic Processes (Fall 2022)

Goals and topics

This is a graduate course on applied stochastic processes, designed for those students who are going to need to use stochastic processes in their research but do not have the measure-theoretic background to take the Math 561-562 sequence. Measure theory is not a prerequisite for this course. However, a basic knowledge of probability theory (Math 461 or its equivalent) is expected, as well as some knowledge of linear algebra and analysis. The goal of this course is a good understanding of basic stochastic processes, in particular discrete-time and continuous-time Markov chains, and their applications. The materials covered in this course include the following:

- **Fundamentals**: background on probability, linear algebra, and set theory.
- **Discrete-time Markov chains**: classes, hitting times, absorption probabilities, recurrence and transience, invariant distribution, limiting distribution, reversibility, ergodic theorem, mixing times;
- **Continuous-time Markov chains**: same topics as above, holding times, explosion, forward/backward Kolmogorov equations;
- **Related topics**: Discrete-time martingales, potential theory, Brownian motion;
- **Applications**: Queueing theory, population biology, MCMC.

This course can be tailored to the interests of the audience.

Weblinks

Course	go.illinois.edu/math564
Grades	Canvas
Discussion	TBA

Logistics

Instructor	Partha Dey
Office	35 CAB
Contact	Email psdey@illinois.edu with subject "Math 564:" (Use your official @illinois.edu address).
Class	TR 9:30am-10:50am in TBA.
Student Hrs	TBA, or by appointment. I will be happy to answer your questions in my office anytime as long as I'm not otherwise engaged.
Textbook	1. **Norris**: *Markov Chains*, Cambridge University Press, 1998;
Other Refs	2. **Levin, Peres, and Wilmer**: *Markov Chains and Mixing Times*, AMS, 2009; 3. **Grimmett and Stirzaker**: *Probability and Random Processes*, 4th Ed., OUP, 2020.
Prerequisite	Math 461 (Undergraduate Probability) and MATH 447/448 (Undergraduate Analysis). A basic knowledge of probability theory, linear algebra and analysis is expected. Measure theory is not a prerequisite for this course.
Grading Policy	**Homework**: 60% of the course grade. Homework problems will be assigned approximately every two weeks. I will post the assigned exercises on Canvas. You are encouraged to work together on the homework, but I ask that you write up your own solutions and turn them in separately. There will be few problems assigned; emphasis will be placed on clear, concise, and coherent writing. **Late homework will not be graded and credited.** **Scribe** (Due one week after class): 10% of the course grade. Scribe lecture notes in LaTeX (using the provided template) for 1 lecture. Again emphasis will be placed on clear, concise, and coherent writing. Latex can be freely downloaded from here. The source LaTeX files for class notes are available below. **Final exam**: 30% will depend on a take home final exam. Take home final exam will be assigned on TBA (last day of instructions) and is due on TBA.

Tentative Timeline

Week		Date	Due	Content
1	T	Aug 23		Set theory and Measure Theory basics. PDF TeX
	R	Aug 25	Homework 0	Probability and Random variables. PDF TeX
2	T	Aug 30		Expectation and Basics of linear algebra. PDF TeX
	R	Sep 01		Definition of Markov chain. PDF TeX
3	T	Sep 06		Properties of Markov chains. PDF TeX
	R	Sep 08	Homework 1	Hitting time and stopping time. PDF TeX
4	T	Sep 13		Strong Markov property. PDF TeX
	R	Sep 15	HW1 Solution	Class structure. PDF TeX
5	T	Sep 20		Recurrence and Transience. PDF TeX
	R	Sep 22	Homework 2	Invariant distribitions. PDF TeX
6	T	Sep 27		Positive recurrence and aperiodicity. PDF TeX
	R	Sep 29	HW2 Solution	Convergence to invariant distribution. PDF TeX
7	T	Oct 04		Convergence for periodic MC. PDF TeX
	R	Oct 06	Homework 3	Time reversal and detailed balance. PDF TeX
8	T	Oct 11		Ergodic theory and Metropolis-Hastings algorithm. PDF TeX
	R	Oct 13	HW3 Solution	Mixing time. PDF TeX
9	T	Oct 18		Continuos Time Markov Chains. PDF TeX
	R	Oct 20	Homework 4	Construction of CTMC. PDF TeX
10	T	Oct 25		Poisson Process. PDF TeX
	R	Oct 27	HW4 Solution	Poisson Process and Birth Processes. PDF TeX
11	T	Nov 01		Explosion time and Minimal Chain. PDF TeX
	R	Nov 03	Homework 5	Class Structure, Hitting Times, Recurrence and Transience PDF TeX
12	T	Nov 08		Martingales. PDF TeX
	R	Nov 10	HW5 Solution	Invariant measure for CTMC. PDF TeX
13	T	Nov 15		Time reversal and convergence to equilibrium. PDF TeX
	R	Nov 17	Homework 6	Martingale characterization. PDF TeX
14	T	Nov 22		No classes. Thanksgiving break.
	R	Nov 24		
15	T	Nov 29		Branching processes. PDF TeX
	R	Dec 01	HW6 Solution	Epedemics and queueing theory. PDF TeX
16	T	Dec 06	Homework 7	Brownian Motion. PDF TeX
	R	Dec 08	HW7 Solution	No Class.

Math 580: Combinatorics; graduate course at UIUC.

General Syllabus

Lectures: *Those note follow Doug West: Combinatorial Mathematics book, in average, one lecture notes covers about one 50 minutes lecture (or maybe a bit more). Note that there is some overlap between consecutive lectures, in order to fit material of one class into one file.*

Lecture I: page1, page2, page 3: Basic counting, Pigeonhole Principle, Words.

Lecture II: Binomial Theorem, Pascal triangle, Basic identities of binomial coefficients.

Lecture III: Delannoy numbers, Cayley formula (number of trees).

Lecture IV: Corollaries of Cayley formula proof; Ballot theorem;

Lecture V: Catalan numbers, Number of rooted binary trees, Number of triangulations, Section 2.1: Fibonacci recurrences, Number of deragements of permutations, basic recursion formulas,

Lecture VI: Number of simple k-words on [n]; numbef of partitions of [n] into k non-empty classes; Number of permutations with k-cycles; 2.2 Section: solution methods;

Lecture VII: Characteristic equations, Tower of Hanoi, Number of regions of plane, Inhomogeneous sequences, Generating function method.

Lecture VIII: Main Theorem of linear recurrences, Section 2.3: Substitution method. Chapter 3.1: Ordinary Generating functions,

Lecture IX: Section 3.1: Ordinary Generating functions, Permutation statistic, Worpitzky Theorem.

Lecture X: Worpitzky Theorem; Section 3.2: OGF Coefficients; Snake Oil;

Lecture XI: Snake Oil, Section 3.3: Exponential Generating functions,

Lecture XII: Stirling number of seconds kind; Stirling number (first kind); Binomial Inversion formula; Exponential Formula, Partition of an n-set; Permutations and Involutions;

Lecture XIII : Number of connected graphs; Exponential Formula; Partitions of an n-set; Permutations and Involutions; Langrange Involuation Formula; Section 3.4: Partitions of Integers;

Lecture XIV : Hardy-Ramanujan formula on number of partitions; Ferrer's diagram; Fallon's formula; Euler's formula; 4.1 Section: Inclusion Exclusion Formula; Stirling's numbers;

Lecture XV: Inclusion Exclusion Formula; Generalized PIE; Disjoint Path Systems; MacMahon Theorem; Section 4.2: Polya Redfield Method;

Lecture XVI: Generalization of derangements; Signed Involutions; Section 4.2: Polya Redfield Method; Burnside Lemma;

Lecture XVII: Determinants, disjoint paths systems; MacMahon Theorem on number of rhombic tilings; Section 4.2: Polya-Redfield counting; Burnside lemma; Number of colorings of the cube.

Lecture XVIII: Burnside lemma; Number of colorings of the cube.

Lecture XIX: Section 4.3: General ballot problem, Chapter 5: Concepts of graphs; Petersen graph,

Lecture XX: Kneser graph, Hypercube, Section 5.2: Handshake lemma, Rectangles partitioned into rectangles with integer sidelengths, Havel-Hakini on degree sequences.

Lecture XXI: Bipartite subgraphs of graphs, Turan Theorem, Directed graphs, Kings in tournaments, Section 5.3: connectivity, Min degree 2 implies having cycle, Eulerian circuits.

Lecture XXII: Cut vertex, cut edge, Eulerian circuits. Section 5.4: Trees, Chapter 6: matchings, Halls Theorem, Hakini Theorem, Birkhoff-Neumann on double stochastic matrices.

Lecture XXIII: Trees, Chapter 6: matchings, Halls Theorem, Hakini Theorem, Birkhoff-Neumann on double stochastic matrices. Defect formula for bipartite graphs, König-Egerváry theorem, Gallai Theorem, König Theorem.

Lecture XXIV: Defect formula for bipartite graphs, König-Egerváry theorem, Gallai Theorem, König Theorem. Section 6.2: Matchings in general graphs, Tutte Theorem, Berge-Tutte formula.

Lecture XXV: Berge-Tutte formula, k-regular multigraph having one-factor, Peterson Theorem, Peterson 2-factor theorem, Section 6.3: Augmenting lines, Section 7.1: Connectivity parameters.

Lecture XXVI: Section 7.1: Connectivity parameters, Whitney Theorem, Blocks, Section 7.2: k-connected graphs, Menger Theorem.

Lecture XXVII: Section 7.2: k-connected graphs, Menger Theorem, Expansion lemma, Dirac Theorem (k points in a cycle in k-connected graphs).

Lecture XXVIII: Fan Lemma, Ford-Fulkerson Common System Distinct Representatives, 2,3-connected graphs, Whitney theorem for ear-decomposition, Robbins Theorem.

Lecture XXIX: Section 7.3: Hamilton cycles, Ore Lemma, Dirac Theorem, Chvatal condition on Hamiltonicity, Chvatal-Erdos condition, Erdos-Gallai on circumference.

Lecture XXX: Erdos-Gallai on circumference, Chapter 8.1: Vertex coloring, Brooks Theorem (no proof), Szekered-Wilf Theorem, Gallai-Roy Theorem, Mycielski construction, Sectionm 8.2: Color-critical graphs.

Lecture XXXI: Mycielski construction, Sectionm 8.2: Color-critical graphs, List coloring, Section 8.3: edge-coloring, König: every bipartite graph is max-degree colorable, Perfect graphs.

Lecture XXXII: Section 8.3: edge-coloring, König: every bipartite graph is max-degree colorable, Perfect graphs. Chapter 9: Planar graphs.

Lecture XXXIII: Chapter 9: Planar graphs. Outerplanar graphs. Euler formula. Characterization of regular polyhedra. Section 9.2: Structure of planar graphs. Section 9.3: Coloring planar graphs, proof of 5-color theorem. Example for applying the discharging method.

Lecture XXXIV : Section 9.2: Structure of planar graphs. Section 9.3: Coloring planar graphs, proof of 5-color theorem. Example for applying the discharging method. Chapter 10.2: Ramsey Theory, probabilistic lower bound on diagonal Ramsey, Inductive proof of general Ramsey theorem (that Ramsey numbers are finite).

Lecture XXXV: Inductive proof of general Ramsey theorem (that Ramsey numbers are finite). Erdős- Szekeres: points in convex position, Chvatal: Ramsey trees vs cliques, Burr- Erdős- Spencer: Ramsey of m vertex disjoint triangles, Section 10.3: Schur Theorem, Chapter 14.1: Erdős: minimum number of edges of non-2-colorable n-uniform hypergraphs, Pluhár, Kozik-Cherkasin lower bounds, Improving diagonal Ramsey lower bounds.

Lecture XXXVI: Chapter 14.1: Erdős: minimum number of edges of non-2-colorable n-uniform hypergraphs, Pluhár, Kozik-Cherkasin lower bounds, Improving diagonal Ramsey lower bounds. Symmetric Erdős-Lovász Local Lemma (statement), application to diagonal Ramsey. Existence of large girth, large chromatic number graphs, Caro-Wei proof of Turán Theorem. Markov's Inequality. Second Moment Method, Chebyshev Inequality. In G(n,p) thresholds for connectivity and having no isolated vertex.

Lecture XXXVII : Caro-Wei proof of Turán Theorem. Markov's Inequality. Second Moment Method, Chebyshev Inequality. In G(n,p) thresholds for connectivity and having no isolated vertex. Chapter 13: Latin squares. Existence of n-1 pairwise orthogonal Latin squares of order n. Projective planes. Construction. Reimann: Constructing bipartite C_4-free graphs. Kӧváry- Sós- Turán: upper bound on the number of edges of bipartite C_4-free graphs.

Lecture XXXVIII: Szemerédi Regularity Lemma [statement only]. Embedding Lemma, Counting Lemma. Chapter 13: Latin squares. Existence of n-1 pairwise orthogonal Latin squares of order n. Projective planes. Construction. Reimann: Constructing bipartite C_4-free graphs. Kövári- Sós- Turán: upper bound on the number of edges of bipartite C_4-free graphs. Block desings.

Lecture XXXIX: Block desings. Fisher's Inequality, Bose theorem, Hadamard matrix, Difference sets.

Lecture XL: Chapter 12: Posets, Dilworth Theorem, Sperner Theorem, LYMB Inequality, Erdõs-Ko-Rado Theorem, Katona circle method, Hoffman bound,

analysis and topology in analytic combinatorics

The course will cover a collection of topics around complex-analytic tools useful for combinatorics.

We will rely heavily on Melszer, Pemantle and Wilson books, but also on material beyond those.
The course will be graded on 3-4 homeworks (50%) and a project (50%).

Topics to be covered:

- Introduction into combinatorial species and generation of generating functions.
- Generating functions and their coefficients. Cauchy formula. One function, many series.
- Dependence on parameters and basic formalism of statistical mechanics. Rudiments of large deviation theory.
- Many variables: amoebas, their properties.
- Theory of residues in higher dimensions. Iterated residues.
- Main results for rational generating function: case of smooth pole.
- Asymptotics of oscillating integrals.
- Lattice quantum random walks.
- Algebraic generating functions.
- Frozen regions in dimer models and rational generating functions with singular poles.
- Petrovsky-Atiyah-Bott-Gårding theory of fundamental solutions to hyperbolic linear PDEs; applications to generating functions.
- Lacunas and discontinuities of the coefficient growth exponents.

* 595 Fukaya Categories of Surfaces

The Fukaya category is a sophisticated invariant of symplectic manifolds whose definition in full generality is intricate. In the original formulation, objects of this category are certain "branes" supported on Lagrangian submanifolds, and the composition laws in the category involve pseudo-holomorphic maps with Lagrangian boundary conditions (Floer theory).

At the same time, a category, like any other algebraic structure, can admit many presentations. For instance, we could try to find a presentation of the Fukaya category by generators and relations, or we could try to build the Fukaya category of a symplectic manifold up from categories associated to smaller pieces of the manifold.

Let S be a oriented surface (2-dimensional real manifold). The Fukaya category of S is now well-understood enough that we know multiple ways of presenting it. The goal of this course is to study the Fukaya category of S via multiple presentations, the original Floer theoretic definition being only one of them. Others include the theory of "gentle algebras" (by results of Haiden-Katzarkov-Kontsevich), and a perspective that views the Fukaya category of S as an object in homological algebra (Dyckerhoff-Kapranov).

Thus, although this course will include some Floer theory, it will not be strictly speaking necessary in order to understand all of the different approaches to the Fukaya category of a surface. My hope is that students with different backgrounds in geometry and algebra will be able to find at least one way of thinking about the Fukaya category that makes sense to them. Applications of these ideas to Homological Mirror Symmetry will also be discussed.

Math 595

Representation-theoretic methods in quantum information theory

Fall term 2022 Lecturer: Felix Leditzky

In this course we study symmetries in quantum information theory using tools from representation theory. Two fundamental symmetry groups in quantum information are the symmetric group, acting by permuting subsystems in a tensor product of identical Hilbert spaces, and the unitary group, acting diagonally on a tensor product space. Schur-Weyl duality establishes a close relationship between these two representations, giving rise to a useful decomposition of the representation space into irreducible representations. This structure result allows us to succinctly describe invariant objects and characterize optimal information-processing protocols in the presence of permutation and unitary symmetries.

The first half of the course starts with a brief review of the basics of quantum information theory and representation theory. We then discuss the representation theory of the symmetric and unitary groups and how they relate to each other via Schur-Weyl duality. These findings can be applied to characterize symmetric quantum states such as Werner and isotropic states.

In the second half of the course we use these representation-theoretic methods to characterize quantum information-processing tasks such as data compression, spectrum estimation, quantum state tomography, and quantum state merging. Depending on available time and interest, we will also discuss useful results such as de Finetti theorems and the decoupling theorem.

Class time

Tue & Thu 02:00–03:20, 147 Altgeld Hall. Office hours by appointment.

Prerequisites

- Required:
  - Math 416 Abstract Linear Algebra (or equivalent)
  - Math 417 Intro to Abstract Algebra (or equivalent)
- Useful but not necessary:
  - Math 506 Group Representation Theory
  - Math 522 Lie Groups and Lie Algebras I
  - Intro to Quantum Mechanics/Information
    (such as ECE 404, Phys 486/487, Phys 513)

Please get in touch at leditzky@illinois.edu if you have any questions on this.

Grading policy

There will be no mandatory homework assignments or written exams for this course. Grading will be based on class participation. I will provide exercises that we can discuss in office hours.

Course outline

- Basics of quantum information theory (review)
- Basics of representation theory (review)
- Representation theory of the symmetric and unitary groups (review)
- Schur-Weyl duality
- Werner states, isotropic states, covariant quantum channels
- Permutation invariance and de Finetti theorems
- Data compression and type theory
- Spectrum estimation and quantum state tomography
- Decoupling theorem and quantum state merging

Literature

- Matthias Christandl. "The structure of bipartite quantum states-insights from group theory and cryptography". Available online. PhD thesis. University of Cambridge, 2006.
- Aram W. Harrow. "The church of the symmetric subspace". *arXiv preprint* (2013). Available online.
- Jean-Pierre Serre. *Linear Representations of Finite Groups*. Graduate Texts in Mathematics. New York: Springer, 1977.
- Constantin Teleman. *Representation Theory*. Lecture notes. Available online. 2005.
- Michael Walter. *Symmetry and Quantum Information*. Lecture notes. Available online. 2018.
- John Watrous. *The Theory of Quantum Information*. Available online. Cambridge: Cambridge University Press, 2018.
- Mark M. Wilde. *Quantum information theory*. 2nd edition. Available online. Cambridge: Cambridge University Press, 2016.

FALL 2022, MATH 595, EXPONENTIAL SUMS

INSTRUCTOR: ALEXANDRU ZAHARESCU

Math 595 ES, TR 9:30 - 10:50PM, room 145 Altgeld Hall

Exponential sums play an important role in many questions in number theory, and also in some problems arising from other fields. The first part of the course will cover classical material. For this part we will follow selected chapters from Montgomery's book. In the second part of the course we will study some recent papers on the distribution of zeros of the Riemann zeta function, points on curves over finite fields, billiards, lattice points and Farey fractions, where exponential sums play a central role.

Prerequisite: MATH 531.

Recommended Textbook:

H. L. Montgomery, *Ten lectures on the interface between analytic number theory and harmonic analysis*, CBMS Regional Conference Series in Mathematics, 84. Providence, RI, 1994.

There will be no exams. Students registered for this course will be expected to give one or two lectures on some topics related to the content of the course. In addition some homework problems will be assigned.

Office hours by appointment.

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