Congruences for r-colored partitions

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Let $\ell \geq 5$ be prime. For the partition function $p(n)$ and $5 \leq \ell \leq 31$, Atkin found a number of examples of primes $Q \geq 5$ such that there exist congruences of the form $p(\ell Q^3 n + \beta) \equiv 0 \pmod{\ell}$. Recently, Ahlgren, Allen, and Tang proved that there are infinitely many such congruences for every ℓ. In this paper, for a wide range of $c \in F_\ell$, we prove congruences of the form $p(\ell Q^3 n + \beta_0) \equiv c \cdot p(\ell Q n + \beta_1) \pmod{\ell}$ for infinitely many primes Q. For a positive integer r, let $p_r(n)$ be the r-colored partition function. Our methods yield similar congruences for $p_r(n)$. In particular, if r is an odd positive integer such that $3r$ for which $\ell > 5r + 19$ and $2^{r+2} \not\equiv 2^{\pm 1} \pmod{\ell}$, then we show that there are infinitely many congruences of the form $p_r(\ell Q^3 n + \beta) \equiv 0 \pmod{\ell}$. Our methods involve the theory of modular Galois representations.