Congruences for r-colored partitions

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Let \( \ell \geq 5 \) be prime. For the partition function \( p(n) \) and \( 5 \leq \ell \leq 31 \), Atkin found a number of examples of primes \( Q \geq 5 \) such that there exist congruences of the form \( p(\ell Q^3n + \beta) \equiv 0 \pmod{\ell} \). Recently, Ahlgren, Allen, and Tang proved that there are infinitely many such congruences for every \( \ell \). In this paper, for a wide range of \( c \in \mathbb{F}_\ell \), we prove congruences of the form \( p(\ell Q^3n + \beta_0) \equiv c \cdot p(\ell Qn + \beta_1) \pmod{\ell} \) for infinitely many primes \( Q \). For a positive integer \( r \), let \( p_r(n) \) be the \( r \)-colored partition function. Our methods yield similar congruences for \( p_r(n) \). In particular, if \( r \) is an odd positive integer such that \( 3r \) for which \( \ell > 5r + 19 \) and \( 2^{r+2} \not\equiv 2^{\pm 1} \pmod{\ell} \), then we show that there are infinitely many congruences of the form \( p_r(\ell Q^3n + \beta) \equiv 0 \pmod{\ell} \). Our methods involve the theory of modular Galois representations.