Course Outline — Combinatorial Mathematics, Math 584, Spring 2022

Professor József Balogh, Classes are MWF: 10:00-10:50, Altgeld 347;
e-mail: jobal@illinois.edu, **Office Hours:** After class and by appointments.
Web page: https://faculty.math.illinois.edu/∼jobal.
Final Exam: 1:30-4:30 p.m., Monday, May 9.

PREREQUISITES: Math 580 or consent of instructor, obtainable by familiarity with elementary combinatorics. Students need the mathematical maturity and background for graduate-level mathematics. For example, basics of linear algebra, probability and graph theory are assumed to be known. Students need to be **independent**.

COURSE REQUIREMENTS: There will be five homework assignments. Homework allows you to choose five out of six problems to write up. Problems are worth 10 points each, so the maximum score/homework is 50 points. Additionally, there will be a final exam for 150 points. A is from 80%, grade drops by 5% (so 75% is A-). Make up possibilities include giving lecture.

TEXT: Jiri Matousek: 33 Miniatures, Mathematical and Algorithmic Applications of Linear Algebra + selected research papers.

TOPICS: The course is about the linear algebraic methods in combinatorics. Recent new breakthrough results should be included. The Linear Algebraic Method is a powerful tool in tackling many problems in discrete mathematics. It belongs to those areas of mathematics which have experienced a most impressive growth in the past few decades. This course provides an extensive treatment of the Linear Algebraic Method, with emphasis on methodology. We will try to illustrate the main ideas by showing the application of the method to various combinatorial problems.

Some of the material to be covered (not necessary in this order):

Algebraic methods: Ray-Chaudhuri-Wilson Theorem Oddtown Theorem, two-distance sets, large t-uniform family with small intersections, Non-uniform modular RW Theorem, application to Ramsey construction, Omitted-Intersection Theorem, unit-distance graph, modular Snevily Theorem, combinatorial nullstellensatz, Cauchy-Davenport Theorem, Chevalley-Warning Theorem, transversal of hyperplanes in cubes, existence of regular subgraphs, ex(n;k-regular), generalized factors, stunted trees, antimagic graphs.

Ramsey theory: pigeonhole principle, Stanley-Wilf conjecture, intro to Schur/Rado/ Van der Waerden's Theorems.

Topological methods: Sperner's Lemma, Brouwer Fixed-Point Theorem, bandwidth equivalence of Sperner, Connector, Hex, Pouzet, Brouwer Tucker's Combinatorial Lemma, Borsuk-Ulam Theorem, Kneser Conjecture, Gale's Lemma, Schrijver graphs, Ham Sandwich and applications disproof of Borsuk's Conjecture.

RESOURCES: Electronic mail is a medium for announcements and questions. I will use the e-mail given in the system, I am not responsible in case it is not monitored. Students are supposed to use illinois.edu e-mails. Some of the communication will be via MOODLE, including homework announcements, and homework submitions.