Math 418, Intro to Abstract Algebra II

Spring 2022

- **Time and Place:** MWF at 11am in 143 Henry.
- **Section:** M13 **CRN:** 38013
- **Instructor:** Nathan Dunfield
  - **E-mail:** nmd@illinois.edu
  - **Office:** 378 Altgeld **Office Phone:** (217) 244-3892
  - **Office Hours:** Monday and Tuesday from 1:30-2:30pm; other times possible by appointment.
  - **Web page:** http://dunfield.info/418/
- **Detailed schedule, including HW and lecture notes**

Course Description

This is a second course in abstract algebra, covering the following topics:

- **Rings:** Polynomial rings, fields of fractions, and other examples. Euclidean domains, principal ideal domains, and unique factorization domains.
- **Fields:** Field extensions and Galois Theory. Solvability of equations by radicals. Ruler and compass constructions.
- **Algebraic geometry:** Basic correspondence between ideals and varieties in affine and projective space, with examples such as elliptic curves. Decomposition into irreducibles, Hilbert's Nullstellensatz, and connections to Galois Theory.

Prerequisites:

The needed background for this course is Math 417, Intro to Abstract Algebra. Math 427 is also fine, though there is some overlap between that course and this one.

Required text: Dummit and Foote, *Abstract Algebra*, 3rd Edition, 944 pages, Wiley 2003. **As of Jan 5, the campus bookstore has a different text listed for this course, but this should be corrected shortly.**

Supplementary texts: For the final part of the course covering algebraic geometry, one good reference beyond Chapter 15 of Dummit and Foote is:

Cox, Little, and O'Shea, *Ideals, Varieties, and Algorithms*, Springer Undergraduate Texts in Mathematics.

You can get it in PDF format via the library's e-book collection. Another nice book is, which is also freely available online is:

Reid, *Undergraduate Algebraic Geometry*.

First week

As per university policy, our first week will be online. **The lectures will be held live rather than be pre-recorded.** I will email everyone enrolled in the class the Zoom link on Tuesday, January 18, in time for my office hour; if you have not yet registered or are auditing, send me an email.

Grading

Your course grade will be based on:

- **Weekly homework assignments:** (20%) These will typically be due on Wednesday. Late homework will not be accepted; however, your lowest two homework grades will be dropped, so you are effectively allowed two infinitely late assignments. Collaboration on homework is permitted, nay encouraged. However, you must write up your solutions individually and *understand them completely*.
- **Two takehome midterms:** (12.5% each) These are glorified HW assignments that you are to work on individually. They will replace the usual HW for two weeks of the term.
- **In class midterm:** (20%) This 50 minute exam will be held in our usual classroom, on Monday, March 9.
- **Final exam:** (35%) This will be Wednesday, May 11 from 1:30-4:30am in our usual classroom.

Schedule

Jan 19 Introduction.
Jan 21 Euclidean Domains.
Jan 24 Principal Ideal Domains.
Jan 26 PIDs are UFDs. **HW 1 due**.

Jan 28 Which polynomial rings are UFDs?

Jan 31 $R[x]$ is a UFD if R is; irreducibility criteria.

Feb 2 Field extensions I. **HW 2 due**.

Feb 4 Field extensions II.

Feb 7 Algebraic numbers and extensions.

Feb 9 More on algebraic extensions. **HW 3 due**.

Feb 11 Field multiplication as linear transformations.

Feb 14 Limitations of straightedge and compass.

Feb 16 Constructible numbers. **Takehome #1 due**;

Feb 18 Splitting fields.

Feb 21 Algebraically closed fields; the Fundamental Theorem of Algebra.
Here are various proofs of the of the Fundamental Theorem of Algebra.

Feb 23 Polynomials with distinct roots; separability criterion. **HW 4 due**.

Feb 25 Finite fields; cyclotomic fields.

Feb 28 Cyclotomic polynomials and applications.

Mar 2 Introduction to Galois Theory. **HW 5 due**.

Mar 4 Galois groups of splitting fields.

Mar 7 **In class midterm.**

Mar 9 Primitive extensions and minimal polynomials.

Mar 11 No class, read about The Fundamental Theorem of Algebra instead.

Mar 12 Spring Break starts.

Mar 20 Spring Break ends.

Mar 21 Finite fields and degrees of fixed fields.

Mar 23 The Fundamental Theorem of Galois Theory I.

Mar 25 The Fundamental Theorem of Galois Theory II. **HW 6 due**.

Mar 28 Possible Galois groups and the discriminant

Mar 30 Galois groups of polynomials.

Apr 1 Solving equations by radicals; solvable groups.

Apr 4 Characterizing solvability by radicals.

Apr 6 Introduction to Algebraic Geometry. **HW 7 due**.

Apr 8 Radical ideals and the Nullstellensatz.

Apr 11 Decomposition into irreducibles and more on Hilbert's results. Also, here is a proof of the Nullstellensatz for arbitrary fields.

Apr 13 Functions on varieties. **HW 8 due**.

Apr 15 Projective space I.

Apr 18 Projective space II.

Apr 20 Elliptic curves. **Takehome #2 due**.

Apr 22 Topology of curves and function fields of varieties.

Apr 25 Rational functions and field extensions.

Apr 27 Rational functions and field extensions II. **HW 9 due**.

Apr 29 Branched covers.

May 2 Cayley graphs and branch covers.

May 4 Branched covers and the Riemann Existence Theorem. **HW 10 due**.