

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF MATHEMATICS
Course Description — FALL 2021

MATH 582

STRUCTURE OF GRAPHS

TOPICS: This is a companion course to Math 581 — Extremal Graph Theory. The two courses are independent. *Structure of Graphs* includes topics drawn from the following (not all will be covered).

Elementary Structural Concepts — structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs. Graph packings and equitable colorings.

The reconstruction problem — is G reconstructible from the deck of subgraphs obtained by deleting a single vertex? ... a single edge?

Connectivity — min-max relations for connectivity and branchings, structure of k -connected graphs.

Cycles — Hamiltonian cycles and circumference in graphs and digraphs.

Topological Graph Theory — embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, proof of Kuratowski's Theorem, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces. Using discharging for coloring problems on surfaces.

Joins and flows — the language of conservative weightings for finding maximal joins and minimum T -joins, cycle covers and nowhere-zero flows.

Graph Minors — treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has infinitely many minimal forbidden minors), forbidden and forced minors.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The problems require proofs related to or applying results from class.

PREREQUISITES: Familiarity with elementary graph theory. Either of Math 580 and Math 412 provides sufficient preparation. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, *Graph Theory*, or the Math 412 text: West, *Introduction to Graph Theory* (Prentice Hall, 2001, first seven chapters). Important results needed from elementary graph theory will be reviewed.

TEXT: D. B. West, *The Art of Combinatorics, Volume II: Structure of Graphs*. For some topics, instructor's supplements will be provided.