University of Illinois at Urbana-Champaign Department of Mathematics

ASRM 406, Linear Algebra and Applications, Fall 2021

SYLLABUS

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Instructor: Joseph Rosenblatt, Department of Mathematics

Text: Linear Algebra and Its Applications, 5th Edition, Pearson, 2016
Authors: Lay, Lay, and McDonald

Content: This course emphasizes techniques of linear algebra. Topics include linear equations, matrix theory, vector spaces, linear transformations, eigenvalues and eigenvectors and inner product spaces. In addition, computational methods and applications to data science, finance, economics, and actuarial science will be explored.

Topics to be Covered: Topic coverage is in line with the text and will proceed during the term in the order of topics in the text. Applications will be given throughout the term, but a review of these will occur at the end of the course. As time allows, topics will include

1. Linear Equations in Linear Algebra
- 1.1 Systems of Linear Equations
- 1.2 Row Reduction and Echelon Forms
- 1.3 Vector Equations
- 1.4 The Matrix Equation Ax = b
- 1.5 Solution Sets of Linear Systems
- 1.6 Applications of Linear Systems
- 1.7 Linear Independence
- 1.8 Introduction to Linear Transformations
- 1.9 The Matrix of a Linear Transformation
- 1.10 Linear Models in Business, Science, and Engineering

2. Matrix Algebra
- 2.1 Matrix Operations
- 2.2 The Inverse of a Matrix
- 2.3 Characterizations of Invertible Matrices
- 2.4 Partitioned Matrices
- 2.5 Matrix Factorizations
- 2.6 The Leontief InputOutput Model
- 2.7 Applications to Computer Graphics
- 2.8 Subspaces of $\mathbb{R}^n$
- 2.9 Dimension and Rank

3. Determinants
- 3.1 Introduction to Determinants
- 3.2 Properties of Determinants
- 3.3 Cramers Rule, Volume, and Linear Transformations

4. Vector Spaces
- 4.1 Vector Spaces and Subspaces
- 4.2 Null Spaces, Column Spaces, and Linear Transformations
- 4.3 Linearly Independent Sets; Bases

4.4 Coordinate Systems
4.5 The Dimension of a Vector Space
4.6 Rank
4.7 Change of Basis
4.8 Applications to Difference Equations
4.9 Applications to Markov Chains

5. Eigenvalues and Eigenvectors

5.1 Eigenvectors and Eigenvalues
5.2 The Characteristic Equation
5.3 Diagonalization
5.4 Eigenvectors and Linear Transformations
5.5 Complex Eigenvalues
5.6 Discrete Dynamical Systems
5.7 Applications to Differential Equations
5.8 Iterative Estimates for Eigenvalues

6. Orthogonality and Least Squares

6.1 Inner Product, Length, and Orthogonality
6.2 Orthogonal Sets
6.3 Orthogonal Projections
6.4 The GramSchmidt Process
6.5 Least-Squares Problems
6.6 Applications to Linear Models
6.7 Inner Product Spaces
6.8 Applications of Inner Product Spaces

7. Symmetric Matrices and Quadratic Forms

7.1 Diagonalization of Symmetric Matrices
7.2 Quadratic Forms
7.3 Constrained Optimization
7.4 The Singular Value Decomposition
7.5 Applications to Image Processing and Statistics

8. The Geometry of Vector Spaces - as time allows

8.1 Affine Combinations
8.2 Affine Independence
8.3 Convex Combinations
8.4 Hyperplanes
8.5 Polytopes
8.6 Curves and Surfaces