

WEIGHT 2 CM NEWFORMS AS P-ADIC LIMITS

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ABSTRACT. Previous works have shown that certain weight 2 newforms are p -adic limits of weakly holomorphic modular forms under repeated application of the U -operator. The proofs of these theorems originally relied on the theory of harmonic Maass forms. Ahlgren and Samart obtained strengthened versions of these results using the theory of holomorphic modular forms. Here, we use such techniques to express all weight 2 CM newforms which are eta quotients as p -adic limits. In particular, we show that these forms are p -adic limits of the derivatives of the Weierstrass mock modular forms associated to their elliptic curves.

1. INTRODUCTION

Suppose that E is an elliptic curve over \mathbb{Q} with Weierstrass model

$$E : y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6.$$

Let N_E be its conductor. By the modularity of elliptic curves over \mathbb{Q} , there exists a lattice Λ_E and a modular parametrization

$$\Phi_E : X_0(N_E) \rightarrow \mathbb{C}/\Lambda_E \cong E.$$

Martin and Ono [MO97] proved that there are five such curves with complex multiplication whose associated newform, $g_{N_E}(z)$, is an eta-quotient. These forms lie in $S_2(N_E)$, the space of weight 2 cusp forms on $\Gamma_0(N_E)$. The following table lists these curves.

N_E	$g_{N_E}(z)$	a_1	a_2	a_3	a_4	a_6	CM field
27	$\eta^2(3z)\eta^2(9z)$	0	0	1	0	-7	$\mathbb{Q}(\sqrt{-3})$
32	$\eta^2(4z)\eta^2(8z)$	0	0	0	4	0	$\mathbb{Q}(i)$
36	$\eta^4(6z)$	0	0	0	0	1	$\mathbb{Q}(\sqrt{-3})$
64	$\frac{\eta^8(8z)}{\eta^2(4z)\eta^2(16z)}$	0	0	0	-4	0	$\mathbb{Q}(i)$
144	$\frac{\eta^{12}(12z)}{\eta^4(6z)\eta^4(24z)}$	0	0	0	0	-1	$\mathbb{Q}(\sqrt{-3})$

TABLE 1. The five curves

For an elliptic curve E over \mathbb{Q} , Alfes, Griffin, Ono, and Rolén [AGOR15] constructed harmonic Maass forms using the Weierstrass ζ -function which comes from its modular parametrization. These forms encode arithmetic information about its Hasse-Weil L -function. The holomorphic parts of these forms are called *Weierstrass mock modular forms*. For each of the above curves, Clemm [Cle16] showed that the derivative of the associated Weierstrass mock modular form is an eta-quotient or a twist of one. The following table lists these forms (where χ_8 and χ_{12} are the Kronecker symbols with discriminants 8 and 12). We have normalized them to have leading coefficient 1.

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N_E	$G_{N_E}(z)$
27	$\frac{\eta(3z)\eta^6(9z)}{\eta^3(27z)}$
32	$\frac{\eta^2(4z)\eta^6(16z)}{\eta^4(32z)}$
36	$\frac{\eta^3(6z)\eta(12z)\eta^3(18z)}{\eta^3(36z)}$
64	$G_{32} \otimes \chi_8$
144	$G_{36} \otimes \chi_{12}$

Write $g_{N_E} = \sum a_{N_E}(n)q^n$ and $G_{N_E} = \sum C_{N_E}(n)q^n$. Using the theory of harmonic Maass forms, Clemm showed that if p is a prime which is inert in the CM field of E and $p \nmid C_{N_E}(p)$, then as a p -adic limit, we have

$$\lim_{m \rightarrow \infty} \frac{G_{N_E}|U(p^{2m+1})}{C_{N_E}(p^{2m+1})} = g_{N_E}. \quad (1.1)$$

In the case where $N_E = 32$, the cusp form g_{32} gives the Hasse-Weil L -function of the congruent number elliptic curve

$$y^2 = x^3 - x.$$

For primes p which are inert in $\mathbb{Q}(i)$ with $p \nmid C_{32}(p)$, El-Guindy and Ono [EGO10] showed (1.1) for $N_E = 32$ using the theory of harmonic Maass forms. Ahlgren and Samart strengthened this result using the theory of holomorphic modular forms. Let $v_p(\cdot)$ denote the p -adic valuation on $\mathbb{Z}[[q]]$. If p is a prime which is inert in $\mathbb{Q}(i)$, they show that for all integers $m \geq 0$, we have

$$v_p(C_{32}(p^{2m+1})) = m, \\ v_p\left(\frac{G_{32}|U(p^{2m+1})}{C_{32}(p^{2m+1})} - g_{32}\right) \geq m + 1.$$

In this paper, we prove the following theorem.

Theorem 1.1. *Let E be one of the elliptic curves in Table 1. Write $g_{N_E} = \sum a_{N_E}(n)q^n$ and $G_{N_E} = \sum C_{N_E}(n)q^n$. Let p be a prime which is inert in the CM field of E . Then for all integers $m \geq 0$, we have*

$$v_p(C_{N_E}(p^{2m+1})) = m, \quad (1.2)$$

$$v_p\left(\frac{G_{N_E}|U(p^{2m+1})}{C_{N_E}(p^{2m+1})} - g_{N_E}\right) \geq m + 1. \quad (1.3)$$

The paper is organized as follows. In Section 2, we give background material on modular forms. In Sections 3, 4, and 5, we prove Theorem 1.1.

2. BACKGROUND

For the next several paragraphs, we follow the exposition in [AS16]. Suppose that $k \in \mathbb{Z}$ and that N is a positive integer. For a function $f(z)$ on the upper half plane and

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}_2^+(\mathbb{Q}),$$

we have the weight k slash operator

$$f(z)|_k \gamma := \det(\gamma)^{\frac{k}{2}} (cz + d)^{-k} f\left(\frac{az + b}{cz + d}\right).$$

We denote by $M_k^!(N)$ the space of forms f which satisfy the transformation law

$$f|_k \gamma = f \quad \text{for} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(N)$$

and which are holomorphic on the upper half plane and meromorphic at the cusps of $\Gamma_0(N)$. We identify each $f \in M_k^!(N)$ with its q -expansion. That is, if $q := \exp(2\pi iz)$, we can write $f(z) = \sum a(n)q^n$ for some coefficients $a(n)$. Let $M_k(N) \subseteq M_k^!(N)$ be the subspace of forms which are holomorphic at the cusps of $\Gamma_0(N)$. Let $M_k^\infty(N) \subseteq M_k^!(N)$ be the subspace of forms which vanish at all cusps of $\Gamma_0(N)$ other than ∞ and $S_k(N) \subseteq M_k^\infty(N)$ be the subspace of forms which vanish at all of the cusps.

We next recall the U and V operators. For a positive integer m , we define them on Fourier expansions by

$$\begin{aligned} \left(\sum a(n)q^n\right) |U_m &:= \sum a(mn)q^n, \\ \left(\sum a(n)q^n\right) |V_m &:= \sum a(n)q^{mn}. \end{aligned}$$

For a positive integer m , let $T_k(m)$ be the usual Hecke operator on $M_k^!(N)$. If p is a prime with $p \nmid N$, $n \geq 1$ is an integer, and $f \in M_k^!(N)$, then

$$F|T_k(p^n) = \sum_{j=0}^n p^{(k-1)j} f|U(p^{n-j})|V(p^j). \quad (2.1)$$

Define

$$\Theta := \frac{1}{2\pi i} \frac{d}{dz} = q \frac{d}{dq}. \quad (2.2)$$

We have the following result (see e.g. [AS16, Lemma 2.1]).

Lemma 2.1. *If $(m, N) = 1$, then*

$$T_k(m) : M_k^\infty(N) \rightarrow M_k^\infty(N). \quad (2.3)$$

If $k \geq 2$, then

$$\Theta^{k-1} : M_{2-k}^\infty(N) \rightarrow M_k^\infty(N). \quad (2.4)$$

Finally, we review some facts about filtrations. If p is a prime such that $p \nmid 6N$ and k is a nonnegative integer, let $M_k^{(p)}(N)$ be the subset of forms in $M_k(N)$ which have coefficients which are p -integral rational numbers. if $f = \sum a(n)q^n \in M_k^{(p)}(N)$, we define

$$\bar{f} := \sum \overline{a(n)}q^n \in \mathbb{F}_p[[q]],$$

and we define

$$\overline{M_k^{(p)}(N)} := \{\bar{f} : f \in M_k^{(p)}(N)\}.$$

If $f \in M_k^{(p)}(N)$, then we define the filtration of \bar{f} as

$$w_p(\bar{f}) = \inf\{k' : \text{there exists } g \in M_{k'}(N) \text{ with } \bar{f} = \bar{g}\}.$$

We make use of the following facts from [Joc82, §7].

Proposition 2.2. *Suppose that $f \in M_k^{(p)}(N)$ and that $w_p(\bar{f}) \neq -\infty$. Then we have*

- (1) $w_p(\bar{f}) \equiv k \pmod{p-1}$.
- (2) $w_p(\overline{f|V(p)}) = pw_p(\bar{f})$.

3. PROOF OF THEOREM 1.1 FOR $N_E = 27$

The proof of Theorem 1.1 for $N_E = 27$ requires proving some preliminary results. Recalling our notation $g_{27} = \sum a_{27}(n)q^n$ and $G_{27} = \sum C_{27}(n)q^n$, we have the following result.

Proposition 3.1. *For every integer $m \geq -1$ except for $m = 0$, there exists a unique form $H_m \in M_2^\infty(27) \cap \mathbb{Z}((q))$ of the form*

$$H_m = q^{-m} + O(q^2).$$

Moreover, if $p \neq 3$ is prime and n is a nonnegative integer, we have

$$G_{27}|T_2(p^n) = p^n H_{p^n} + C_{27}(p^n)g_{27}.$$

Proof. Define

$$L_1(z) = \frac{\eta^4(9z)}{\eta(3z)\eta^3(27z)} = q^{-2} + q + O(q^4),$$

$$L_2(z) = \frac{\eta^3(3z)}{\eta^3(27z)} = q^{-3} - 3 + O(q^3).$$

For integers $d \geq 0$, consider the forms

$$g_{27}(z)L_1^d(z) = q^{-2d+1} + O(q^{-2d+4}), \quad (3.1)$$

$$g_{27}(z)L_1^d(z)L_2(z) = q^{-2d} + O(q^{-2d+3}). \quad (3.2)$$

The fact that the forms L_1 and L_2 are holomorphic at all of the cusps other than ∞ (see e.g. [Ono04, Thm 1.64, Thm 1.65]) implies that the forms in (3.1) and (3.2) are in $M_2^\infty(27)$. By taking linear combinations of these forms, we obtain the forms H_m . Since the space $S_2(27)$ is one-dimensional, the forms H_m are unique. Some examples of these forms are given below.

$$H_{-1} = g_{27} = q - 2q^4 + \dots,$$

$$H_1 = G_{27} = q^{-1} - q^2 + \dots,$$

$$H_2 = q^{-2} - 5q^4 + \dots.$$

To prove the last part of the proposition, note that

$$G_{27}|T_2(p^n) = G_{27}|U(p^n) + \sum_{j=1}^{n-1} p^j G_{27}|U(p^{n-j})|V(p^j) + p^n G_{27}|V(p^n)$$

by (2.1). By (2.3), we have $G_{27}|T_2(p^n) \in M_2^\infty(27)$. Since

$$G_{27} = q^{-1} - q^2 + \dots,$$

we have

$$G_{27}|U(p^n) = C_{27}(p^n)g_{27} + O(q^2)$$

and

$$\sum_{j=1}^{n-1} p^j G_{27}|U(p^{n-j})|V(p^j) + p^n G_{27}|V(p^n) = p^n q^{-p^n} + O(q^2).$$

This implies that

$$G_{27}|T_2(p^n) - p^n H_{p^n} \in S_2(27).$$

Since $S_2(27)$ is one-dimensional, we have $G_{27}|T_2(p^n) - p^n H_{p^n} = C_{27}(p^n)g_{27}$. \square

Before we prove Theorem 1.1 for $N_E = 27$, we require the following congruence.

Lemma 3.2. *For each prime p which is inert in $\mathbb{Q}(\sqrt{-3})$ and each integer $m \geq 0$, we have the congruence*

$$C_{27}(p^{2m+1}) \equiv (-1)^m p^m C_{27}(p) \pmod{p^{m+1}}.$$

Proof. By definition, L_1 has an expansion of the form

$$L_1(z) = \sum b(n)q^{3n+1} \in \mathbb{Z}((q)),$$

and L_2 has an expansion of the form

$$L_2(z) = \sum c(n)q^{3n} \in \mathbb{Z}((q)).$$

Suppose that $p \equiv 2 \pmod{3}$. By taking polynomials in L_1 and L_2 with integer coefficients, we can construct a modular function $\psi_p \in M_0(27)$ which is holomorphic at all of the cusps of $\Gamma_0(27)$ other than ∞ of the form

$$\psi_p(z) = q^{-p} + C_p q + O(q^4). \quad (3.3)$$

Since $G_{27} \in M_2^\infty(27)$, we see that the meromorphic differential $G_{27}(z)\psi_p(z)dz$ on the modular curve $X_0(27)$ is holomorphic at all of the cusps of $X_0(27)$ other than ∞ . The sum of the residues of a meromorphic differential is zero, and the residue at ∞ is a multiple of the constant term. Since the constant term of $G(z)\psi_p(z)$ is $C_p + C_{27}(p)$, we have $C_p = -C_{27}(p)$.

By Lemma 2.1, we have

$$\Theta(\psi_p) = -pq^{-p} - C_{27}(p)q + O(q^4) \in M_2^\infty(27). \quad (3.4)$$

By Proposition 3.1, we also have

$$-G_{27}|T_2(p) = -pq^{-p} - C_{27}(p)q + O(q^4).$$

Thus,

$$G_{27}|T_2(p) = -\Theta(\psi_p), \quad (3.5)$$

which means that

$$G_{27}|U(p) = -\Theta(\psi_p) - pG_{27}|V(p). \quad (3.6)$$

By applying $U(p^2)$ to both sides of (3.6), an induction argument implies for each $m \geq 0$ that

$$G_{27}|U(p^{2m+1}) = \sum_{k=0}^m (-1)^{m-k+1} p^{m-k} \Theta(\psi_p)|U(p^{2k}) + (-1)^{m+1} p^{m+1} G_{27}|V(p).$$

The fact that $\Theta(\psi_p)|U(p^{2k}) \equiv 0 \pmod{p^{2k}}$ implies for each integer $m \geq 0$ that

$$G_{27}|U(p^{2m+1}) \equiv (-1)^{m+1} p^m \Theta(\psi_p) \pmod{p^{m+1}}. \quad (3.7)$$

The result follows by comparing the coefficients of q in (3.7) using (3.4). \square

The next lemma helps us to establish (1.2).

Lemma 3.3. *For each prime p which is inert in $\mathbb{Q}(\sqrt{-3})$, we have $p \nmid C_{27}(p)$.*

Proof. Assume for the sake of a contradiction that $p \mid C_{27}(p)$. From Proposition 3.1 and (3.5), we see that

$$\Theta(\psi_p) = -G_{27}|T_2(p) = -pH_p - C_{27}(p)g_{27} \equiv 0 \pmod{p}.$$

This implies that ψ_p is of the form

$$\psi_p \equiv q^{-p} + \sum_{n=1}^{\infty} A(pn)q^{pn} \pmod{p}$$

for some integral coefficients $A(pn)$. A computation in Magma of an integral basis for $M_2(27)$ gives

$$f_1 = q + \cdots, \quad f_2 = q^2 + \cdots, \quad f_3 = q^3 + \cdots, \quad f_4 = q^4 + \cdots, \quad f_5 = q^6 + \cdots. \quad (3.8)$$

Set $h_p = \psi_p f_5^p \in M_{2p}(27)$. For some integral coefficients $D(pn)$, we have

$$h_p \equiv \sum_{n=5}^{\infty} D(pn)q^{pn} \equiv q^{5p} + \cdots \pmod{p},$$

which means that

$$h_p \equiv h_p|U(p)|V(p) \pmod{p}.$$

Since

$$h_p|U(p) \equiv h_p|T(p) \pmod{p},$$

we have $\overline{h_p|U(p)} \in \overline{M_2^{(p)}(27)}$. By (2) of Proposition 2.2, we have $w_p(\overline{h_p}) = pw_p(\overline{h_p|U(p)})$ and $w_p(\overline{h_p}) \equiv 2p \pmod{p-1}$. The fact that $p \mid w_p(\overline{h_p})$ implies that $w_p(\overline{h_p}) = 2p$, so we have $w_p(\overline{h_p|U(p)}) = 2$. Thus, there exists a form $h_0 \in M_2^{(p)}(27)$ with

$$h_0 \equiv h_p|U(p) \equiv q^5 + O(q^6) \pmod{p}.$$

However, an examination of the above basis for $M_2(27)$ shows that no such form h_0 exists. The result follows. \square

Proof of Theorem 1.1 for $N_E = 27$. Lemmas 3.2 and 3.3 imply (1.2). To prove (1.3), note that Proposition 3.1 and (2.1) give

$$\frac{G_{27}|U(p^{2m+1})}{C_{27}(p^{2m+1})} - g_{27} = \frac{1}{C_{27}(p^{2m+1})} \left(p^{2m+1}H_{p^{2m+1}} - \sum_{j=1}^{2m+1} p^j G_{27}|U(p^{2m+1-j})|V(p^j) \right). \quad (3.9)$$

By (2.1), we have

$$G_{27}|T_2(p^{2m}) = \sum_{j=1}^{2m+1} p^{j-1} G_{27}|U(p^{2m+1-j})|V(p^{j-1}).$$

By the definition of G_{27} , we see that $C_{27}(n) = 0$ whenever $n \not\equiv 2 \pmod{3}$. Proposition 3.1 then implies that $G_{27}|T_2(p^{2m}) = p^{2m}H_{p^{2m}}$. Thus, we have

$$\sum_{j=1}^{2m+1} p^j G_{27}|U(p^{2m+1-j})|U(p^{2m+1-j})|V(p^j) = p^{2m+1}H_{p^{2m}}|V(p) \equiv 0 \pmod{p^{2m+1}}. \quad (3.10)$$

From (3.10), we see that (1.2) and (3.9) give (1.3). \square

4. PROOF OF THEOREM 1.1 FOR $N_E = 36$

The proof of Theorem 1.1 for $N_E = 36$ is similar to the proof for $N_E = 27$; fewer details will be given. We have the following analogue of Proposition 3.1 above.

Proposition 4.1. *Suppose that $m \geq -1$ is an odd integer. There exists a unique form $H_m \in M_2^\infty(36) \cap \mathbb{Z}((q))$ of the form*

$$H_m = q^{-m} + O(q^3).$$

For nonnegative integers n and primes $p \geq 5$, we have

$$G_{36}|T_2(p^n) = p^n H_{p^n} + C_{36}(p^n)g_{36}.$$

Proof. Define

$$L(z) := \frac{\eta(6z)\eta^3(9z)}{\eta(3z)\eta^3(18z)}.$$

For integers $d \geq 0$, consider the forms

$$g_{36}(z)L^d(2z) = q^{-2d+1} + O(q^{-2d+7}). \quad (4.1)$$

Since $L(2z)$ is holomorphic at all of the cusps of $\Gamma_0(36)$ other than ∞ , the forms in (4.1) are in $M_2^\infty(36)$. These forms are supported on odd exponents; it follows that we can take linear combinations to get forms H_m with the desired properties. These forms are unique because the space $S_2(36)$ is one-dimensional. The proof of the last assertion follows as in the proof of Proposition 3.1. \square

We now prove an analogue of Lemma 3.2

Lemma 4.2. *For each prime $p \geq 5$ which is inert in $\mathbb{Q}(\sqrt{-3})$ and each integer $m \geq 0$, we have the congruence*

$$C_{36}(p^{2m+1}) \equiv (-1)^m p^m C_{36}(p) \pmod{p^{m+1}}.$$

Proof. Define

$$\psi_2(z) = L(2z) = \sum b(n)q^{6n+4} = q^{-2} + O(q^4) \in \mathbb{Z}((q)),$$

$$\psi_3(z) = L(z)L(2z) - 1 = \sum c(n)q^{6n+3} = q^{-3} + O(q^3) \in \mathbb{Z}((q)).$$

Suppose that $p \equiv 5 \pmod{6}$. Then, we can construct a modular function which is supported on exponents r with $r \equiv 1 \pmod{6}$ of the form

$$\psi_p = q^{-p} - C_{36}(p)q + O(q^7).$$

This implies that

$$\Theta(\psi_p) = -pq^{-p} - C_{36}(p)q + O(q^7) \in M_2^\infty(36).$$

By Proposition 4.1, we also have

$$-G_{36}|T_2(p) = -pq^{-p} - C_{36}(p)q + O(q^7).$$

Thus,

$$G_{36}|T_2(p) = -\Theta(\psi_p). \quad (4.2)$$

The proof follows as in the proof of Lemma 3.2. \square

We now prove an analogue of Lemma 3.3.

Lemma 4.3. *For each prime $p \geq 5$ which is inert in $\mathbb{Q}(\sqrt{-3})$, we have $p \nmid C_{36}(p)$.*

Proof. Suppose by way of contradiction that $p \equiv 5 \pmod{6}$ has $p \mid C_{36}(p)$. By Proposition 4.1 and (4.2), we have

$$\Theta(\psi_p) \equiv 0 \pmod{p}.$$

This implies that there exist integral coefficients $A(pn)$ such that

$$\psi_p \equiv q^{-p} + \sum_{n=1}^{\infty} A(pn)q^{pn} \pmod{p}.$$

A computation in Magma of an integral basis for $M_2(36)$ reveals that there is a form $f \in M_2(36) \cap \mathbb{Z}[[q]]$ with

$$f = q^{12} + \dots.$$

Set $h_p = \psi_p f^p \in M_{2p}(36)$. For some integral coefficients $D(pn)$, we have

$$h_p \equiv \sum_{n=11}^{\infty} D(pn)q^{pn} \equiv q^{11p} + \dots \pmod{p},$$

which means that

$$h_p \equiv h_p|U(p)|V(p) \pmod{p}.$$

As in Lemma 3.3, we conclude that there exists a form $h_0 \in M_2^{(p)}(36)$ with

$$h_0 \equiv h_p|U(p) \equiv q^{11} + O(q^{12}) \pmod{p}.$$

However, an examination of a basis for $M_2(36)$ implies that no such form h_0 exists. The result follows. \square

The proof of Theorem 1.1 for $N_E = 36$ follows as in the proof for $N_E = 27$.

5. PROOFS OF THEOREM 1.1 FOR $N_E = 64$ AND $N_E = 144$

Theorem 1.1 for $N_E = 64$ and $N_E = 144$ follow from the cases $N_E = 32$ and $N_E = 36$. In particular, if χ_8 and χ_{12} are the Kronecker characters with discriminants 8 and 12, we have

$$g_{64} = g_{32} \otimes \chi_8, \quad G_{64} = G_{32} \otimes \chi_8,$$

$$g_{144} = g_{36} \otimes \chi_{12}, \quad G_{144} = G_{36} \otimes \chi_{12}.$$

The result follows from using, for example, the relationship

$$(G_{32} \otimes \chi_8)|U(p^{2m+1}) = \chi_8(p^{2m+1})(g|U(p^{2m+1}) \otimes \chi_8).$$

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