Course Descriptions

Fall 2021

Department of Mathematics
University of Illinois

C O U R S E D E S C R I P T I O N

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FALL 2021
MATH 502

INTRODUCTION TO COMMUTATIVE ALGEBRA
Professor S. P. Dutta
9:30 – 10:50 Tu-Th

This course is intended mainly for students who are going to specialize in Commutative Algebra, Algebraic Geometry, Algebraic K-theory and Algebraic Number Theory.
In this course we will mainly focus on Noetherian rings and modules. The topics will include: Primary decomposition, Artin-Rees Lemma, Flatness, Completion, Hilbert-Samuel Polynomial, Dimension Theory, Integral extensions, Going-up and Going-down theorems, Noether's Normalization (its geometric interpretation), Regular rings and the notion of depth. We would also like to study Cohen-Macaulayness if time permits.

Prerequisite: Math500, 501
Recommended text: Commutative ring theory by H. Matsumura

Course description

In this course we will study the theory of Riemann surfaces. Riemann surfaces provides basic examples for algebraic as well as complex geometry. We will aim to cover the theorems of Riemann-Hurwitz and Riemann-Roch, classification of differential equations, abelian integrals, the theorem of Abel-Jacobi and the construction of Jacobian varieties. Time permitting, we will also talk about vector bundles on Rieman surfaces.

Prerequisites: abstract algebra, complex analysis, familiarity with differential or algebraic geometry.

MATH 512
ALGEBRAIC GEOMETRY I
FALL 2021

Time: TuTh 11:00–12:15
Instructor: Sheldon Katz
Text: Algebraic Geometry, R. Hartshorne, Graduate Texts in Mathematics 52, Springer NY 1977
Supplementary text: The Rising Sea. Foundations of Algebraic Geometry. R. Vakil, online course notes
Prerequisites: Abstract Algebra II (Math 501), or Introduction to Algebraic Geometry (Math 511), or permission of the instructor

This course introduces Algebraic Geometry from both the algebraic and geometric viewpoints. Affine and projective varieties will be developed from both the classical viewpoint (quickly) as well as using the language of schemes and sheaves.

The course will focus on the first two chapters of the main text: Varieties, and Schemes, primarily the second chapter on Schemes, developing the theory of schemes, sheaves, divisors, and their properties. As time allows, we will cover applications such as the theory of curves.

The text will be frequently supplemented with additional materials designed to enhance geometric intuition and teach students by working through examples how to *do* algebraic geometry.

Math 518

Professor Tolman

Definitions and properties of differentiable manifolds and maps, (co)tangent bundles, vector fields and flows, Frobenius theorem, differential forms, exterior derivatives, integration and Stokes' theorem, DeRham cohomology, inverse function theorem, Sard's theorem, transversality and intersection theory. Prerequisite: MATH 423 or MATH 481, or consent of instructor.

Math 520

Professor Baryshnikov

This course presents an introduction to the fundamentals of symplectic geometry and some applications. It is intended for PhD students studying symplectic geometry, Poisson geometry, and symplectic topology, as well as students in related areas such as dynamical systems, algebraic geometry, complex geometry, low dimensional topology and mathematical physics. The course covers the local and global structure of symplectic manifolds, their submanifolds, the special automorphisms they support (Hamiltonian flows), their natural boundaries (contact manifolds), their special geometric features (almost complex structures), and their symmetries and a range of special topics (quasiclassical asymptotics and Lagrangean singularities, applications in optimization and control, …)

Students taking this course are assumed to know differential geometry at the level of Math 518 – Differentiable Manifolds.

Textbooks:

- Ana Cannas da Silva, ***Lectures on Symplectic Geometry***, Springer, 2008.
- Dusa McDuff and Dietmar Salamon, ***Introduction to Symplectic Topology***, Oxford graduate texts in mathematics 27, OUP, 2017.

Grading: Homework assignments (about 6, 60%) + project (report on a paper, or an exposition of a research topic, 40%).

Weekly plan, list of project topics etc, – TBA.

Math 526, Algebraic Topology II

Fall 2021

- **Time and Place:** MWF from 2:00-2:50pm in TBA.
- **Section:** M1 **CRN:** 59521
- **Instructor:** Nathan Dunfield
  - **E-mail:** nmd@illinois.edu
  - **Office:** 378 Altgeld. **Office Phone:** (217) 244-3892
  - **Office Hours:** TBA.
- **Web page:** http://dunfield.info/526
- **Homework assignments**
- **Lecture notes**

Course Description

Math 526 is a second course in algebraic topology. It develops the theory of cohomology, which is homology's algebraically dual sibling, and applies it to a wide range of geometric problems. A key advantage of cohomology over homology is that it has a multiplication, called the cup product, which makes it into a ring; for manifolds, this product corresponds to the exterior multiplication of differential forms. The course includes the study of Poincaré duality which interrelates the (co)homology of a given manifold in different dimensions, as well as topics such as the Kunneth formula and the universal coefficient theorem.

The other major topic covered in this course are the higher homotopy groups, including things like cellular approximation, Whitehead's theorem, excision, the Hurewicz Theorem, Eilenberg-MacLane spaces, and representability of cohomology.

Basically, we'll cover Chapters 3-4 of the required text, which is

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002. ISBN: 0521795400

You can download the full text for free here. A useful list of errata is also available. Two other helpful perspectives on this material, which are quite different from both Hatcher and each other, are:

- Bott and Tu, *Differential forms in algebraic topology*, Springer GTM #82.
- J. Peter May, *A Concise Course in Algebraic Topology*, revised edition.

Additional topics covered will depend on audience interest, but may include spectral sequences, basic sheaf cohomology, and homology with local coefficients. Sources here include

- Davis and Kirk, *Lecture Notes in Algebraic Topology*.
- Hatcher, *Spectral Sequences in Algebraic Topology*.
- Hatcher, *Vector Bundles and K-Theory*.

Prerequisites: Math 525 and Math 500 or similar.

Grading

Your course grade will be based on:

- **Homework assignments:** These will be roughly biweekly (so 7 assignments in total), and typically due on Wednesdays. Late homework will not be accepted; however, your lowest two homework grades will be dropped so you are effectively allowed two infinitely late assignments. Collaboration on homework is permitted, nay encouraged. However, you must write up your solutions individually.
- **In-class participation:** This includes attendance and contributing to class discussion.

Homework Assignments

- TBA.

Lecture notes

Here are scans of my lecture notes from the last time I taught this course (2014), I will update them as we go along.

1. Aug 23. Introduction. Covered pages 1-4.
2. Aug 25. Cohomology: examples and properties. Covered pages 1-4 and 5 through the statement of the general UTC.
3. Aug 27. The Universal Coefficient Theorem. Did pages 1-5.
4. Aug 30. The UCT part II; Intro to the cup product on cohomology. Did pages 1-5 and also 6 very quickly.
5. Sept 1. Definition of the cup product, basic examples, some properties. Tom's version.
6. Sept 3. More on the cup product. Did pages 1-3 plus the homological example at the top of page 4.
7. Sept 6. Labor Day.
8. Sept 8. Cohomology of product spaces. Covered everything.
9. Sept 10. Cohomology of product spaces II. Did pages 1-(middle of 5) and 7. There are some errors in these notes, see the correction in the next lecture.
10. Sept 13. Application of cohomology to division algebras. Covered everything.
11. Sept 15. Homology of manifolds. Covered everything.
12. Sept 17 Orientations and homology. Pages 1-4 excepting the outline of the proof of the lemma.
13. Sept 20. Orientations and homology: proof of the key lemma. All except uniqueness part of (b) in step 4 on page 5.
14. Sept 22. The statement of Poincaré Duality and the cap product. Everything but the middle block on page 4.
15. Sept 24. Dual cell structure to a triangulation. Covered everything.
16. Sept 27. First proof of Poincaré Duality. Through first paragraph on page 4.
17. Sept 29. Consequences of duality; cohomology with compact support. Through top of page 5.
18. Oct 1. Direct limits and the general duality isomorphism. Covered everything.
19. Oct 4. Second proof of Poincaré Duality. Did pages 1-4.
20. Oct 6. Other forms of duality. Covered everything.
21. Oct 8. Higher homotopy groups.
22. Oct 11. Relative homotopy groups. Through the top section of page 5.
23. Oct 13. More on relative homotopy groups. Did 1-4, though page 4 was rushed.
24. Oct 15. Whitehead's Theorem.
25. Oct 18. Applications of excision. Did 1-4.
26. Oct 20. Proof of excision. Did through middle of 4.
27. Oct 22. Rest of proof of excision; intro to Eilenberg-MacLane spaces. Did 1-top of 5.
28. Oct 25. More on Eilenberg-MacLane spaces. Everything except the proof of the lemma.
29. Oct 27. Homology and homotopy: the Hurewicz homomorphism. Did everything.
30. Oct 29. Fibrations and fiber bundles. Did everything except the proof of local triviality for the Hopf bundle.
31. Nov 1. Fiber bundles. Did pages 1-4.
32. Nov 3. Stable homotopy groups. Did everything.
33. Nov 5. Cohomology via $K(G, n)$'s. Did 1-4.
34. Nov 8. Loopspaces and Ω-spectra. Did 1-4.
35. Nov 10. Ω-spectra give cohomology theories. Did everything.
36. Nov 12. Representability of cohomology. Did through top of page 4. The argument in the middle of page 3 is incomplete, see next lecture.
37. Nov 15. Fiber bundles with structure group. Did 1-middle of 4.
38. Nov 17. More on fiber bundles Did 1-3 with a bit of 5.
39. Nov 19. Homology with local coefficients. Did through very top of 5.
40. Nov 29. Bordism and homology. Did 1-4.
41. Dec 1. Framed bordism and the Pontryagin-Thom construction. Did 1-4.
42. Dec 3. Stably framed bordism and the stable homotopy groups of spheres.
43. Dec 6. TBA/slack.
44. Dec 8. TBA/slack.

Math 531

Professor Thorner

Problems in number theory treated by methods of analysis; arithmetic functions, Dirichlet series, Riemann zeta function, L-functions, Dirichlet's theorem on primes in progressions, the prime number theorem. Prerequisite: MATH 448 and either MATH 417 or MATH 453.

Math 535

Professor Lerman

* Definition and examples of topology, topological spaces and continuous maps, bases, subbases.
* subspaces, products
* metrics and pseudometrics
* quotient topology
* nets
* separation axioms: Hausdorff, regular, normal...
* connectedness, local connectedness, path connectedness
* compactness, Tychonoff theorem
* compactness and completeness in metric spaces
* Urysohn lemma, Tietze extension
* countability axioms
* paracompactness and partitions of unity
* metrizability
* topology on function spaces
* categories, functors and natural transformations
* fundamental groupoid
* covering spaces

COURSE DESCRIPTION FOR MATH 540

XIAOCHUN LI

This course is mainly about the theory of functions on $\mathbb{R}^n$, and it is designed for first-year graduate students in mathematics. The following topics are planed to be covered.

- Abstract measure theory, Lebesgue measure, and measurable functions
- Lebesgue theory of integration and convergence theorems
- Differentiation of functions, functions of bounded variation
- L^p spaces
- Hilbert spaces and Fourier series (if time permits)

Lectures: MWF 11:00–11:50am in 441 Altgeld.

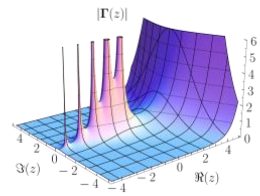
Textbook: G. B. Folland, Real Analysis, John Wiley & Sons

Grading: Homework (15%), 1 or 2 midterms (40%), and a final (45%). No make-up exams.

Exams:The midterm exam will be a 50-minute exam in class. The final will be a 3-hour exam.

Date: February 27, 2020.

Math 542 Complex Variables - Fall 2021



Credit: Wikipedia on “Zeros and poles”

A dry and factual overview could assert that complex analysis investigates analytic and geometric properties of differentiable functions of a complex variable.

That statement would do no justice whatsoever to the power or beauty of the subject. Would you believe that if a complex function can be differentiated once, it can be differentiated infinitely often? That adding a point at infinity to the plane and wrapping up to a sphere enables us to maps circles and lines to each other simply by rotating (stereographic projection)? That a huge number of integrals can be evaluated if you know the behavior of the integrand at just a few special points and loop around them in the correct way (residues)? That the simplest formula for sine is an infinite product? That the right generalization of a line to three dimensions is not a plane but rather a saddle (harmonic functions)? And more…

If you prefer less excitement than this, you may consult instead the list of topics in the official departmental syllabus.

Prerequisites: either Math 446 and 447, or else Math 448.
In practice this means you must have previously studied undergraduate real analysis, and that some undergraduate complex analysis is highly recommended too.
Students who possess plenty of mathematical maturity (e.g. have passed some comps already and know topology) can succeed in Math 542 without having previously taken complex analysis, though they will find the course moves quickly. Otherwise, it would be better to take Math 446 or 448 instead.

Assessment:
Three midterms and a final exam. Weekly homework. Attendance.

Style of the course
Supportive and inclusive. I will run weekly collaborative homework sessions, and want you to come to my office hours and work with other students in the course.

Textbook
Bruce Palka, *An Introduction to Complex Function Theory*
(the yellow cover “Undergraduate Texts in Mathematics” edition from Springer)

Optional reading (for enjoyment and elucidation)
Tristan Needham, *Visual Complex Analysis*, 2nd edition.
His book presents a wonderfully geometric approach to the subject.

- Prof. Richard Laugesen Laugesen@illinois.edu (please email me with questions.)

COURSE DESCRIPTION FOR MATH 550, Fall 2021

Vadim Zharnitsky

This course will mainly focus on the classical theory of dynamical systems. The main part of the course covers 550 comprehensive examination material:

Continuous systems: Phase space, vector fields, flows. Cauchy-Peano existence theorem. Continuous dependence on initial conditions and parameters.

Introduction to discrete dynamics: Iteration of maps, fixed points and stability, chaotic behavior, Bernoulli shift, cat map.

Linear differential equations: Real and complex Jordan normal forms, stability of linear systems with constant coefficients, linearization.

Other topics will include:

- **Geometric methods:** Limit sets and asymptotic behavior, periodic orbits and Poincaré-Bendixson theorem, Floquet theory
- **Nonlinear systems near equilibrium:** Linearization, Hartman-Grobman Theorem, stable manifold theorem.
- **Structural stability:** Smale's horseshoe, hyperbolic systems.
- **Bifurcation theory:** Center manifold theorem, saddle-node bifurcation, pitchfork bifurcation, Hopf bifurcation.
- **Hamiltonian systems:** (If time permits.)

Textbook: Lecture notes based on several texts.

Grading: Homework (15 %), 1 or 2 midterms (40 %), and the final exam (45 %).

Math 554. Linear Analysis and Partial Differential Equations.
Fall 2021

Instructor: Nikos Tzirakis

Phone: (217) 244-8233
Email: tzirakis@illinois.edu
Mail Box: 250 Altgeld Hall

Course Description:

The course covers and develops techniques from functional analysis along with their implementation in the theory of partial differential equations. Time permitting, we will cover the following topics:

• Short introduction to Banach and Hilbert space theory. Banach Fixed Point Theorem and applications to differential and integral equations.

• The theory of L^p spaces. Completeness, Duality, Reflexivity, Convolutions and Mollification, along with the basic inequalities that we use in PDEs.

• Introduction to the spectral theory of bounded linear operators. The spectrum of compact operators, Fredholm Alternative and applications to Layer Potentials. Introduction to the class of unbounded operators. Criteria for self-adjointness.

• Distributions and Fourier Transform on R^n. Laplace transform for initial and boundary value problems. L^2-based Sobolev spaces, Fundamental Solutions, Green's Functions. Heat, Schrödinger or Linear Wave equation on R^n by inverting the Fourier transform.

• Weak derivatives and Sobolev spaces on bounded and unbounded domains. Embedding Theorems, Rellich-Kondrachov Theorem. Elliptic regularity. Lax-Milgram and applications to linear evolution equations (nonconstant coefficients). Galerkin approximation. Examples on Elliptic, Parabolic and Hyperbolic equations.

• Introduction to nonlinear partial differential equations.

Prerequisites: Math 447, Math 489, or consent of instructor. Math 540 would be useful but is not required.

The course will be based on my lecture notes. The notes will be detailed, and self-contained.

Recommended texts:
1. L. E. Evans, Partial Differential Equations.
2. M. B. Erdogan and N. Tzirakis: Dispersive Partial Differential Equations: Wellposedness and Applications.

For students which are not familiar with modern analysis techniques it may be helpful to consult from time to time the *Real Analysis* book of G. B. Folland.

The grade will be based on regular homework, participation and attendance.

Math 562- Professor Dey

Topics:

This is the second half of the basic graduate course in probability theory. The goal of this course is to understand the basic theory of stochastic calculus. We will cover the following topics:

(1) Brownian motion;
(2) Continuous time martingales;
(3) Markov processes;
(4) Stochastic Integrals;
(5) Ito's formula;
(6) Representation of martingales;
(7) Girsanov theorem;
(8) Stochastic Differential equations and
(9) Connections to partial differential equations.

If there is interest from the audience and if time allows, we will also cover applications to mathematical finance.

Prerequisite:

1. Math 561 Probability Theory I - is a prerequisite for this course. However, if you have not taken Math 561, but are willing to invest some extra time to pick up the necessary materials from 561, you may register for this course.
2. Math 540 Real Analysis I - we will review measure theory topics as needed.
3. Math 541 is also nice to have, but not necessary.

Grading:

60% of your grade will depend on homework assignment, and 40% will depend on a take home final exam.

Text:

J. Le Gall: Brownian Motion, Martingales, and Stochastic Calculus, Springer, 2016.

Other references:

1. D. Revuz and M. Yor: Continuous martingales and Brownian motion (3rd edition), Springer, 1999;
2. I. Karatzas and S. E. Shreve: Brownian motion and stochastic calculus (2nd edition), Springer, 1991;
3. S. E. Shreve: Stochastic calculus for finance I and II, Springer, 2004.

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Math 580

Professor Balogh

Syllabus: This is a rigorous, graduate level introduction to combinatorics. It does not assume prior study, but requires mathematical maturity; it moves at a fast pace. The first half of the course is on enumeration. The second half covers graph theory. There are some topics that are treated more in depth in advanced graduate courses (Math 581, 582, 583, 584, 585): Ramsey theory, partially ordered sets, the probabilistic method and combinatorial designs (as time permits).

Textbook: Combinatorial Mathematics, Doug West, Cambridge U. Press, 2020

REQUIREMENTS: A raw score of 80% or higher guarantees an A while a score of 60% or higher guarantees a B- (grade drops by 5%). Additionally, for an A, in the final exam minimum 50%, for B+ (passing com) 40% required. (Near) weekly assignments. Each assignment will have 6 problems of your choice of 5/6 are graded.
There are 9 homework assignments, each worth 4.44%, two tests, each 15%, and a final exam for 30%.
The gradings: 80%− : A, 75%− : A−, 70%− : B+, 65%− : B, etc.
Note that the writings of the solutions must have a high quality and typed, if the argument is messy or not typed then even if the solution is correct it could be returned without grading with 0 points.

Late homework policy: In case the homework is not submitted on time, it could be submitted for the next class, with losing 10% of the score. If there is offical or medical reason then try to notify me in advance via e-mail.
RESOURCES: Electronic mail is a medium for announcements and questions.
PREREQUISITES: Students in the Math doctorate (Ph.D) program can register, even they may want to check if they have sufficient backgrounds and want to take Math 412 or Math 413 first.
For all others, Math 412 and Math 413 (or equivalent), with grades A (and not A-) needed. Computer Science or Engineer Ph.D students should contact the instructor.

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF MATHEMATICS
Course Description — FALL 2021

MATH 582

STRUCTURE OF GRAPHS

TOPICS: This is a companion course to Math 581 — Extremal Graph Theory. The two courses are independent. *Structure of Graphs* includes topics are drawn from the following (not all will be covered).

Elementary Structural Concepts — structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs. Graph packings and equitable colorings.

The reconstruction problem — is G reconstructible from the deck of subgraphs obtained by deleting a single vertex? ... a single edge?

Connectivity — min-max relations for connectivity and branchings, structure of k-connected graphs.

Cycles — Hamiltonian cycles and circumference in graphs and digraphs.

Topological Graph Theory — embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, proof of Kuratowski's Theorem, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces. Using discharging for coloring problems on surfaces.

Joins and flows — the language of conservative weightings for finding maximal joins and minimum T-joins, cycle covers and nowhere-zero flows.

Graph Minors — treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has infinitely many minimal forbidden minors), forbidden and forced minors.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The problems require proofs related to or applying results from class.

PREREQUISITES: Familiarity with elementary graph theory. Either of Math 580 and Math 412 provides sufficient preparation. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, Graph Theory, or the Math 412 text: West, Introduction to Graph Theory (Prentice Hall, 2001, first seven chapters). Important results needed from elementary graph theory will be reviewed.

TEXT: D. B. West, The Art of Combinatorics, Volume II: Structure of Graphs. For some topics, instructor's supplements will be provided.

Math 585

Professor Balogh

The Probabilistic Method is a powerful tool in tackling many problems in discrete mathematics.
It belongs to those areas of mathematics which have experienced a most impressive growth in the past few decades.
This course provides an extensive treatment of the Probabilistic Method, with emphasis on methodology. We will try to illustrate the main ideas by showing the application of probabilistic reasoning to various combinatorial problems. The topics covered in the class will include (but are not limited to):
Linearity of expectation, the second moment method, the local lemma, correlation inequalities, martingales, large deviation inequalities, Janson and Talagrand inequalities, pseudorandomness, random graphs, random regular graphs, percolation, bootstrap percolation.
TEXTBOOKS: Most of the topics covered in the course appear in the following book:
Other topics appear in recent papers.
The Probabilistic Method, by N. Alon and J. H. Spencer, 4th Edition, Wiley,

There will be a test during the semester, and several homework assignments.
GRADING: 80%− : A, . . . , 50% − C−.

OFFICE HOURS: After classes and by appointment.
PREREQUISITES: Official prerequisites is a B+ or better grade in Math 580 class. In case lack of it, please contact the instructor.

Math 595 AT3, Algorithmic topology and geometry of 3-manifolds: theory and practice

Fall 2021

- **Time and Place:** MWF from 11:00-11:50am in TBA. Second half of semester only, starts Oct 18.
- **Section:** AT3 **CRN:** 57742
- **Instructor:** Nathan Dunfield
  - **E-mail:** nmd@illinois.edu
  - **Office:** 378 Altgeld. **Office Phone:** (217) 244-3892
  - **Office Hours:** TBA.
- **Web page:** http://dunfield.info/595B
- **Lecture notes**

Course Description

In dimensions four and higher, most basic questions about manifolds (e.g. is a given manifold the n-sphere?) are algorithmically undecidable. In contrast, many questions about 3-manifolds are not just decidable but have practical algorithms that have been implemented and run on literal millions of 3-manifolds. I will survey some of what is known here, focusing on the use of geometry to solve topological problems in the spirit of Thurston.

Topics will include basics of 3-dimensional topology and the Geometrization Theorem, solvability of the word and homeomorphism problems for 3-manifolds, and verified computation using interval arithmetic to compute hyperbolic structures on 3-manifolds. The exact mix of topics will depend on students' background and interests, but to get the general flavor, see the notes, references, and handouts from a summer school course I taught in 2017. The course is independent from my other 595 course this term and I will minimize the overlap with Eric Sampterton's course from Spring 2021.

Prerequisites: Basic knowledge of smooth manifolds and algebraic topology, e.g. Math 518 and Math 525. No prior knowledge of 3-manifolds will be assumed, but at least a vague interest in computation is recommended.

Grading

Students registered for the course will need to write a short (2-4 page) paper which will be due on Friday, December 9th. This paper is largely free-form, and can be about any subject related to the content of this course. For instance, it could be a brief account of a result not covered in class, a review of the some related results explaining why they are interesting, a detailed work-out of a proof only sketched in class, or careful solutions to problems from class or taken from our various readings. Alternatively, code and/or computations can be substituted for the paper.

Lecture notes

Notes from each lecture will be posted here.

1. Oct 18. First day of class!

Math 595 FTM, Foliations and the topology of 3-manifolds

Fall 2021

- **Time and Place:** MWF from 11:00-11:50am in TBA. First half of semester only, ends Oct 15.
- **Section:** FTM **CRN:** 57739
- **Instructor:** Nathan Dunfield
  - **E-mail:** nmd@illinois.edu
  - **Office:** 378 Altgeld. **Office Phone:** (217) 244-3892
  - **Office Hours:** TBA.
- **Web page:** http://dunfield.info/595A
- **Lecture notes**

Course Description

As with many areas of topology and geometry, the study of 3-dimensional manifolds begins with its codimension-1 submanifolds, namely embedded surfaces. From there, it is very natural to consider foliations of 3-manifolds by surfaces, that is, locally trivial decompositions into (typically noncompact) surface leaves. While every closed orientable 3-manifold has a foliation, the seemingly mild condition of requiring the foliation to be taut restricts both the topology of the ambient manifold (e.g. the universal cover must be $\mathbf{R}^3$) and of the leaves, and forces the existences of interesting transverse dynamics.

I will start with an overview of foliations, focusing on examples, and then discuss the basic properties of taut foliations on 3-manifolds. Then I will then turn to the foundational work of Gabai, specifically his tool of sutured manifold hierarchies for constructing foliations. Next, I will discuss connections to contact geometry, namely the work of Eliashberg-Thurston which builds from a taut foliation a pair of tight contact structures. Finally, I will explain how taut foliations fit into the various Floer homology theories of 3-manifolds (the Heegaard, instanton, and monopole Floer homologies).

Prerequisites: Basic knowledge of smooth manifolds and algebraic topology, e.g. Math 518 and Math 525. No prior knowledge of foliations or 3-manifolds will be assumed.

References

I will not follow any particular text, but recommend the following:

- A. Candel and L. Conlon, *Foliations I*, Graduate Studies in Mathematics **23**, AMS, 1999.
- A. Candel and L. Conlon, *Foliations II*, Graduate Studies in Mathematics **60**, AMS, 2003.
- D. Calegari, *Foliations and the geometry of 3-manifolds*, Oxford Univ. Press, 2007.

Grading

Students registered for the course will need to write a short (2-4 page) paper which will be due on Friday, October 15th. This paper is largely free-form, and can be about any subject related to the content of this course. For instance, it could be a brief account of a result not covered in class, a review of the some related results explaining why they are interesting, a detailed work-out of a proof only sketched in class, or careful solutions to problems from class or taken from one of the texts or other readings.

Lecture notes

Notes from each lecture will be posted here.

1. Aug 23. First day of class!

Math 595 – Professor Fernandes

Lie Groupoids and Lie Algebroids -

This course is an introduction to the theory of Lie groupoids and their infinitesimal counterparts, called Lie algebroids. This is a far reaching extension of the usual Lie theory, which finds application in many areas of Mathematics.
Groups typically arise as the symmetries of some given object. The concept of a groupoid allows for more general symmetries, acting on a collection of objects rather than just a single one. Groupoid elements may be pictured as arrows from a source object to a target object, and two such arrows can be composed if and only if the second arrow starts where the first arrow ends. Just as Lie groups (as introduced by Lie around 1900) describe smooth symmetries of an object, Lie groupoids (as introduced by Ehresmann in the late 1950's) describe smooth symmetries of a smooth family of objects. That is, the collection of arrows is a manifold G, the set of objects is a manifold M, and all the structure maps of the groupoid are smooth. Ehresmann's original work was motivated by applications to differential equations. Since then, Lie groupoids have appeared in many other branches of mathematics and physics. These include:

- algebraic geometry: Grothendieck introduced stacks in the late 1960's via fibered categories over a site. Fibered categories can be viewed as a type of sheaf of groupoids. More recently, this has led to the concept of a gerbe.
- foliation theory: Haefliger introduced transversal structures to foliations in the 1970's, using the concept of a holonomy groupoid. This approach allows for a systematic study of transversal structures, and has been central to the subsequent development of the subject.
- noncommutative geometry and index theory: Lie groupoids made their appearance in noncommutative geometry through the monumental work of Connes in the 1980's. He introduced the tangent groupoid of a space as a central ingredient in his approach to the Atiyah-Singer index theorem. This approach led to a number of refinements of the index theorem, such as the Connes-Skandalis index theory for foliations.
- Poisson geometry: motivated by quantization problems, Karasev and Weinstein introduced the symplectic groupoid of a Poisson manifold in the late 1980's, as a way to ``untwist'' the complicated behavior of the symplectic foliation underlying the Poisson manifold.

In this course I will be following my Lecture Notes:

- M. Crainic and R.L. Fernandes, Lectures on Integrability of Lie Brackets also available as *Geometry & Topology Monographs* **17** (2011) 1-107.

Students taking this course are assumed to know differential geometry at the level of Math 518 - Differentiable Manifolds. A knowledge of ordinary Lie Theorey at the level of Math 522 is recommended but not strictly necessary.

Textbooks:

I will provide to participants some lecture notes as the course progresses, but the following two references should also be very helpful:

- A. Cannas da Silva and A. Weinstein, ***Geometric models for noncommutative algebras***, Berkeley Mathematics Lecture Notes, 10. American Mathematical Society, Providence, RI, 1999.
- M. Crainic and R.L. Fernandes, Lectures on Integrability of Lie Brackets, available as *Geometry & Topology Monographs* **17** (2011) 1-107.
- D. Metzler, Topological and Smooth Stacks, Preprint arXiv:math/0306176.
- A. Vistoli, Grothendieck topologies, fibered categories and descent theory, Fundamental algebraic geometry, 1104, Math. Surveys Monogr., 123, AMS Providence, RI (2005).

Grading Policy

- **Expository Paper:** Students will be encouraged to write (in LaTeX) and present a paper. This is not mandatory. Following the tradition of topics courses, there will be no homework and no written exams.

Math 595: Functional and geometric inequalities

Daesung Kim

Functional and geometric inequalities play an important role in several research areas such as calculus of variations, PDEs, harmonic analysis, probability, convex geometry, and so on. In particular, there have been of great interest in studying sharp forms and quantitative forms of such inequalities. In general, functional and geometric inequalities can be formulated as

$$\mathcal{G}(u) \geq c\mathcal{H}(u)$$

where $\mathcal{G}$ and $\mathcal{H}$ are nonnegative functional on a class of admissible functions (or sets) and c is a positive constant. For example, the Sobolev inequality is the case when $\mathcal{G}(u) = \|\nabla u\|_2$ and $\mathcal{H}(u) = \|u\|_2$. There are three fundamental questions regarding functional and geometric inequalities:

(i) Is the inequality *sharp*? Namely, can we find a constant c satisfying $\mathcal{G}(u) \geq c\mathcal{H}(u)$ such that for all $\lambda > c$, there exists u_λ such that $\mathcal{G}(u_\lambda) < \lambda\mathcal{H}(u_\lambda)$? Such constant is called the sharp constant.

(ii) Once the sharp constant is obtained, can we chracterize the equality cases? In other words, the question is to find a collection of u, say X_0 such that $\mathcal{G}(u) = c\mathcal{H}(u)$ if and only if $u \in X_0$. Such u is called an optimizer.

(iii) After identifying the class of optimizers, a natural question is whether a function u is close to the class of optimizers when u almost attains the equality. This question can be answered by a quantitative version of the inequality: a lower bound of the deficit $\delta(u) = \mathcal{G}(u) - \mathcal{H}(u)$ in terms of a distance between u and the class of the optimizers.

In this course, we discuss the three questions for several inequalities such as the classical isoperimetric inequality, Sobolev inequalities, Hardy–Littlewood–Sobolev inequality, log Sobolev inequality, and some singular integral operators.

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DISTRIBUTION OF SEQUENCES IN NUMBER THEORY

ALEXANDRU ZAHARESCU

Fall 2021, Math 595

The topic has an established history, starting with the distribution of prime numbers and continuing with the study of a large variety of sequences which appear naturally in problems in number theory. Several developments in this area occurred in recent years. In this course we will study some of these results, starting with a review of the theory of uniform distribution of sequences and moving to the finer structure of the distribution of a sequence captured by the level spacing measures and correlation measures. We then proceed to a detailed presentation in various contexts, such as distribution in finite fields, fractional parts of polynomial sequences, Farey fractions, visible points, Ford circles, distribution of zeros of the Riemann zeta-function and more general L-functions.

We will not follow any particular textbook. Instead, we will present material from various papers. There will be no exams. Some homework problems may be assigned. In addition, students enrolled in this class will be expected to give a couple of lectures on topics related to the content of the course.

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