Flat Littlewood Polynomials Exist

A polynomial $P(z) = \sum_{k=0}^{n} \varepsilon_k z^k$ is a Littlewood polynomial if $\varepsilon_k \in \{-1, 1\}$ for all k. Littlewood proved many beautiful theorems about these polynomials over his long life, and in his 1968 monograph he stated several influential conjectures about them. One of the most famous of these was inspired by a question of Erdős, who asked in 1957 whether there exist "flat" Littlewood polynomials of degree n, that is, such that

$$\delta\sqrt{n} \leqslant |P(z)| \leqslant \Delta\sqrt{n}$$

for all $z \in \mathbb{C}$ with $|z| = 1$, for some absolute constants $\Delta > \delta > 0$. In this talk we will describe a proof that flat Littlewood polynomials of degree n exist for all $n \geqslant 2$. The proof is entirely combinatorial, and uses probabilistic ideas from discrepancy theory.

Joint work with Paul Balister, Béla Bollobás, Julian Sahasrabudhe and Marius Tiba.