Math 519 – Differentiable Manifolds II

Spring 2021

This course is the second part of a sequence of two courses dedicated to the study of differentiable manifolds. In the first course we have seen the basic elements of the theory. This course continues with this study and it is divided into two parts: the first part is dedicated to the study of Riemannian manifolds (manifolds with a smooth varying inner product on the tangent spaces); the second part concentrates on more advanced concepts (e.g., vector bundles, principal bundles, connections, etc.), aiming at a deeper understanding of smooth manifolds and geometric structures on them.

If you have not taken Math 518 the past semester, you should make yourself familiar with the topics study in that course (please see the web page of Math 518).

Instructor: Rui Loja Fernandes

Department of Mathematics

Contact Information

- E-mail: ruiloja@illinois.edu
- Office Location: 346 Illini Hall (not in use this semester)
- Office Phone: 217-300-2431 (leave message)
- Office Hours (via zoom): Tue 11:00-11.50AM:

For information about the instructor see the instructor's homepage

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Course Overview

- **Riemannian geometry.** Riemannian metrics, covariant derivatives, parallel transport, geodesics, Hopf-Rinow Theorem. Curvature tensors, first and second variation formulas, Jacobi fields, Myers' and Hadamard's Theorems, Gauss-Bonet Theorem. Hodge-star operator, Laplace operator, harmonica forms, Hodge-de Rham theorem.
- **Fiber bundles.** Vector bundles, principal bundles, connections, parallel transport, curvature, Chern-Weyl theory, Poincaré-Hopf Theorem.
- **Advanced topics:** Either symplectic geometry or Morse theory (time permitting)

Course Location and Time: This course will be fully on-line. Lectures (Zoom sessions) will be held on TTh 9.30-10.50 am CST.

NOTE: Course lectures and discussion sessions will be recorded and posted on-line.

Office hours: Office hours via zoom will be held on Tue at 11 am, following the day's lecture.

Course Goals

The main goals of this course are:

- Understand what are Riemannian metrics, their basic properties and how they are used in differential geometry.
- Learn the basics of bundle theory and its relevance to differential geometry.
- Apply metrics and bundle theory to study some important global invariants of manifolds.

Recommended Textbooks

I will provide some Lecture Notes that cover the 2nd part of the semester (fiber bundles). The recommend textbooks are:

- S. Gallot, D. Hulin and J. Lafontaine, Riemannian Geometry third edition. Springer-Verlag, Universitytext 2004.
- J. Jost, Riemannian Geometry and Geometric Analysis 6th Edition, Springer-Verlag, Universitytext 2011.
- S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, vol 1 and 2 New York, Interscience Publishers, 1963.
- C. Taubes, Differential Geometry: Bundles, Connections, Metrics and Curvature Oxford University Press, 2011.

Background material

For background on algebraic topology see, e.g.:

- William Fulton, Algebraic Topology: A First Course, Springer-Verlag, GTM vol 153, 1995.
- Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.

Homework Assignments

There will be 5 homework assignments. Homework assignments are due by midnight (Central time) on the dates specified in the weekly overviews unless otherwise noted.

If you need an extension on an assignment because of medical reasons or personal emergencies, you must address the issue with the course instructor. Such accommodations will be made on a case-by-case basis.

Homework assignments will be reviewed and graded by the course TA within 1 week. The midterm and final exam will be graded by the course instructor within 5 business days. If your instructor is unable to meet this timeline, students will be notified.

Grades

The final grade will be based on the grade of the 5 homework assignments.