Math 500

Professor Dutta

Basic facts about groups, rings, vector spaces, such as those covered in Math 416 and Math 417 courses, are assumed. Instructors should not spend time on elementary material: the syllabus is quite full. Books that could be used include "Abstract Algebra" by Dummit and Foote, "Algebra" by Hungerford, and "Advanced Modern Algebra" by Rotman.

1. Group Theory. [Approximately 4.5 weeks]
(a) Isomorphism theorems for groups.
(b) Group actions on sets; orbits, stabilizers. Application to conjugacy classes, centralizers, normalizers.
(c) The class equation with application to finite p-groups and the simplicity of A_5.
(d) Composition series in a group. Refinement Theorem and Jordan-Hölder Theorem. Solvable and nilpotent groups.
(e) Sylow Theorems and applications.

2. Commutative rings and Modules.[Approximately 5 weeks]
(a) Review of subrings, ideals and quotient rings. Integral domains and fields. Polynomial rings over a commutative ring.
(b) Euclidean rings, PID's, UFD's.
(c) Brief introduction to modules (over commutative rings), submodules, quotient modules.
(d) Free modules, invariance of rank. Torsion modules, torsion free modules. Primary decomposition theorem for torsion modules over PID's.
(e) Structure theorem for nitely generated modules over a PID. Application to finitely generated Abelian groups and to canonical form of matrices.
(f) Zorn's lemma and Axiom of Choice (no proofs). Application to maximal ideals, bases of vector spaces.

3. Field Theory. [Approximately 5 weeks]
(a) Prime fields, characteristic of a field.
(b) Algebraic and transcendental extensions, degree of an extension. Irreducible polynomial of an algebraic element.
(c) Normal extensions and splitting fields. Galois group of an extension.
(d) Algebraic closure, existence and uniqueness via Zorn's Lemma. Finite fields.
(e) Fundamental theorem of Galois theory.
(f) Examples of Galois extensions. Cyclotomic extensions.
(g) If time permits, application of Galois theory to solution of polynomial equations, symmetric functions and ruler and compass constructions.