Math 496 F1H (Tu Th 9:30-10:45, online) – Course Organization, Spring 2021

Instructor: Prof. Bruce Reznick, 327 Altgeld Hall, 333–4284, reznick@illinois.edu. My phone has voice mail and I frequently check and reply to my email, including weekends. Zoom office hours are by appointment. I take them seriously, and they can usually be arranged within 24 hours. You are also encouraged to ask me questions (especially) during and after class. I'm terrible with names; don't take it personally.

This course will have a Moodle page, which will contain links to all recorded zoom meetings (at least for a month. There will also be a links page.

http://faculty.math.illinois.edu/∼reznick/S21links.html

The Blurb: Few prospects are as daunting for a serious math major as that of doing research. In fact, mathematical research is a natural extension of homework in mathematics courses, except that there is no back of the book to look for the answer. This course is designed to help students develop their skills in mathematical creativity and problem-solving (these skills are useful in all advanced mathematics classes as well.) The only formal prerequisite is Math 347, or the ability to convince the instructor that you can write proofs correctly. However, this **is** an honors course in terms of the approach to the subject.

Some of the ways in which research differs from homework are these: the problems are harder, you may not know whether you (or anyone else) can solve them, you do not always know what you need to know in order to solve them, they are usually motivated by a larger set of questions and, ultimately, by the researcher's own curiosity. Research is also often a matter of synthesizing several seemingly different results into a cohesive whole.

The Course: The best way for a student to become successful in mathematical research is to take as many challenging and meaningful mathematics courses as possible, in many different areas. This course, on the other hand, concentrates on building an infrastructure for research. In the first part, we consider problem-solving, question-asking, answer-analyzing and knowledge-finding, and you will choose a project and start working on it. (The instructor's own Ph.D. thesis began as an undergraduate project, but he cannot guarantee this outcome!) Mathematical creativity is a subset of human creativity, and much is known about how to become more creative. In the second part, students present their own research projects and listen to and critique the work of the others. One of the most important goals of this course is for you to develop the habit of reflecting on mathematical knowledge, internalizing its content and rearranging it into something new.

Format: This course has been offered for more than 20 years, always MWF 2, but to accommodate students who are living in different time zones, the hours this year have been changed. Since it is very difficult to do the online equivalent of the classical chalk talk at the board, I will send you instructions on how to make your presentations in LaTeX, using the beamer format. (If you are proficient already at Powerpoint, that's fine to use, but I won't be able to answer any technical questions about it.)

Since this class is a seminar, you will be expected to participate actively *and attend nearly every class meeting.* Please let me know in advance if you can't come. The standard "unit of talk" for a student is about fifteen minutes, including time for questions, except for the final presentation, which is half an hour.

Typically, I will say something at the beginning of class, either by describing a resource which will help you to do research, or by giving you some actual mathematics I think you might enjoy and showing how research techniques can be applied to the problems. These talks are often motivated by the presentations you make. I hope there will be at least one student presentation each day. I will postpone any presentation I have planned if lots of you get excited about something mathematically and want to share it with the group. During the first few weeks, you'll be asked to make at least one presentation on a mathematical topic you find interesting. By mid-March, you will have selected your research topic, and you'll be asked to talk about it in the middle of the semester. Finally, you'll be asked to talk about your progress by the end of the semester. There will also be other chances to speak.

Some of you may be doing research this semester as an independent study or in the IGL. You can't use the same paper in two courses and you can't use the same project! Your project here has to be different: either be an entirely different topic, or a clearly different aspect of your ongoing work.

Much of the value of the course will come from going through the process of choosing what to investigate! I don't want to choose your problem for you. It's OK to work in a related area (e.g. computer science, economics, physics, statistics), as long as the research has a serious mathematical component. **Collaboration is both acceptable and strongly encouraged, and is good practice for the "real" research you'll do later.**

There will be students at varying levels of mathematical knowledge and sophistication in this class. Don't worry that you don't know enough mathematics — nobody ever knows enough mathematics. (Your professors are continually learning new mathematics.)

Two suggestions from past participants and me: (i) Keep a "discovery notebook" to record your ideas throughout the semester – this will *not* be collected; (ii) Turn in a draft of your final report at least a week before the semester ends, so that I can make detailed suggestions for your final report. Be assertive in giving me feedback on the way things are going: my ambition is that this be the most successful course in the history of undergraduate mathematics, and I'll settle for 90% of that. Previous students have found this course valuable. Become an active participant – let it get under your skin and visit your dreams. These are serious steps towards becoming a mathematician.

The Texts: An article based on the first few weeks of Math 496 appeared in *Math Horizons* a while back. I've passed out the printed version; here is a link to the original submission: https://faculty.math.illinois.edu/∼reznick/mhori.pdf

Two texts are recommended: "Mathematics and plausible reasoning: induction and analogy in mathematics" by George Pólya and "Proofs from the Book" by Martin Aigler and Gunter Ziegler. The first book is part of a classic series of books on problem solving by the man who coined the word "heuristics". The second is a collection of short, accessible and beautiful proofs on a range of mathematical topics. Think of it as the Louvre. These books provide potential points of embarkation for your projects. Two other books which you might enjoy are "The man who loved only numbers" by Paul Hoffman, a biography of Paul Erdös, and "A mathematician's apology" by G. H. Hardy, a thoughtful essay on mathematics. There will be many links to articles which are generally useful or which amplify the

presentations in class. handouts; I distribute articles which amplify the presentations made in class. I take requests on topics.

Homework, Exam (what exam?) and Grading Policy: I will attempt a range of assignments in the first half of the semester. I expect you to concentrate your energies on your projects for the second half of the semester. The final project will be due at the time of the final exam, which is Tue. May 11, 1:30-4:30 pm. If the virus allows, there will be some sort of final party.

For a course such as this, I have no objection to giving very high grades overall, provided they reflect your effort and commitment to the class. Let me repeat: I will take your background into account when evaluating your work, and the more you put in to this course, the more you will take away.

Acknowledgment of power and responsibility: One of the first books I read about education pointed out that a good teacher must "be friendly without being a friend", and so I should discuss some uncomfortable issues explicitly. The professor/student relationship is inherently asymmetric. One person must formally evaluate the other's work, and there are typically major disparities in age and experience; only one of us here looks like the professor in a cartoon. Friends don't grade friends. Nonetheless, cordiality and mutual respect must prevail.

I should say that, although I have never had to defend myself against racism or misogyny or other group-related bigotry, I have some experience at being "othered" by my classmates.

Fifty-one years ago last Fall, I started college at Caltech. I was 16. I looked like I was 12. I **do** know what it's like to have the people in the room look at you like you don't belong. I **do** know what it's like to be bullied. Both are awful. I will tolerate none of that in Math 496. Everyone here (and I include myself) must treat everyone else here with kindness and respect. **Let me know if this fails to happen.**

A final word. Mathematics honors classes are sometimes infected with what might be called "mathismo". The best mathematicians I know have been modest in terms of self-promotion and generous in listening to others. It is a good ideafor you to assume that everyone in this class is an excellent mathematician who might have talents and knowledge in different directions than yours, and might be able to help you with your project. As is true more generally in life, you learn more by listening than by talking.

The success of each of you is equally and personally important to me. I will do what I can to help you achieve it. I can't wait for the semester to begin.