Math 416, Abstract Linear Algebra (Spring 2021)

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Logistics, or how will the course run:
The class will be carried out entirely online and will have synchronous and asynchronous components.

I will provide lecture notes and videos of the lectures. You should also read the corresponding section of the textbook. After watching each video you will have an easy one question quiz to answer in Moodle. This quiz will expire at the time of the lecture.

There will be weekly homework that you will turn in using Moodle. During the class time, we will meet via Zoom (the link is in the course Moodle. I will provide problems for us to work in small groups. Part of your grade will be participation in the synchronous classes.

We will also have midterms and a final. The midterms will be during the class time and I will proctor them via Zoom.

There will be three midterms but we will drop the lowest score. The midterms will be:
1) March 1 (covering hws 1-3),
2) March 29 (covering hws 4-7),
3) April 26 (covering hws 8-11),

The course will have a Campuswire page where I encourage you to post and answer questions. You should get an email invitation to join the class Campuswire page, if you haven't please email me to let me know.

Grading:
Quizzes (15%)
Class participation (20%)
Homeworks (30%)
Midterms (20%)
Final (20%)
(Note that this adds up to 105, so there are 5 extra points built in.)
Everybody has something come up during the semester that can make it hard to keep up with the class. This is especially true in the current crisis. For this reason we will drop the two lowest homework scores, the two lowest quiz scores, and the lowest midterm score.

Text: Meckes & Meckes, Linear Algebra.

 Holidays: Classes begin on January 25 and end on May 5. There will be no classes on:
February 17, March 24, and April 13.

Brief syllabus:
(For a more detailed description of what the course is about, see Lecture 1.)

Linear systems of equations show up in all areas of science and mathematics. One reason is that if something (a map or a space) is not linear then one of our best tools for understanding it is to approximate it by a linear version. For this to be useful we need to have a good understanding of the linear versions. That's what we aim for in this course.

We will cover all sections of the textbook. The chapters are:
1) Linear systems and vector spaces
Solve linear systems of equations leads to the definition of general vector spaces over arbitrary fields.
2) Linear Maps and Matrices
It is a truism in math that to understand a structure you need to study the maps that preserve that structure. So having defined
vector spaces we next study those maps between vector spaces that preserve their linear structure.

3) Linear independence, Bases, and Coordinates
Now we delve deeper into the structure of a vector space and learn how to recognize when two vector spaces are really just different representations of the same space.

4) Inner Products
There are some vector spaces where it makes sense to say that two vectors are orthogonal to each other. This is extra structure so in keeping with our paradigm we also study the maps that preserve this structure.

5) Singular Value Decomposition and the Spectral Theorem
If the same vector space can present itself in ways that can look different, and we are studying a map between vector spaces, we can ask ourselves if one of these ways of describing the vector space(s) makes the map easier to understand. The answer is yes.

6) Determinants
The determinant is a single number that tells us whether or not a map is invertible. It's important to know about and a useful theoretical tool but, except for small matrices, we will usually prefer to use other tools.