

# Fifty Golden Years with Ramanujan

**BRUCE BERNDT** in Conversation with Atul Dixit.



Public lecture titled 'Living with Ramanujan for 40+ years' at IIT Gandhinagar in 2019.

COURTESY: IIT Gandhinagar

**B**ruce Carl Berndt, Professor Emeritus at the University of Illinois at Urbana-Champaign, first came in contact with Ramanujan's famous Notebooks in 1974 while working at the Institute for Advanced Study in Princeton. Realizing that a theorem he had proven a couple of years before could be used to prove certain results from the Notebooks, and not having seen the Notebooks until then, his curiosity got the better of him, to irrevocably draw him towards Ramanujan – a devoted association that continues to this day. Berndt travelled to India several times over, even met Ramanujan's wife Janaki, drew inspiration holding the very slate on which Ramanujan worked out his wonderful theorems, and researched deeply into all aspects concerning his life and work. Engaging with a large number of students, Berndt has so far published five voluminous works in an effort to supply proofs to, as per his count, the 3254 theorems contained in the Notebooks.

Atul Dixit, himself a former PhD student of Bruce Berndt, met him during a conference organized to jointly celebrate the 85th birthdays of Berndt as well as his friend and collaborator George Andrews, attributed with the discovery of the "Lost Notebook" of Ramanujan. In a face-to-face conversation with Atul Dixit, Bruce Berndt journeyed backwards in time, recounted significant moments and events of his life – more than half of it spent in his ongoing search of the enigma of Ramanujan.

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Atul Dixit ([adixit@iitgn.ac.in](mailto:adixit@iitgn.ac.in)), is an Associate Professor of Mathematics at the Indian Institute of Technology, Gandhinagar, and a Swarnajayanti Fellow instituted by Anusandhan National Research Foundation of the Government of India. With an inclination to work in the areas of mathematics developed by Srinivasa Ramanujan, his research area includes special functions, analytic number theory, q-series and the theory of partitions.

I would like to first thank you for agreeing, and for taking the time out, to have your interview taken during this conference. Let me start by asking about your childhood. So could you please begin with a background of yours?

BB: Lately, I've become more interested in my ancestral background, partly because my wife Helen, in the last two or three years, has been digging deeply into her background and has traced her family back on her mother's side to a ship that arrived from England in 1631, eleven years after the Mayflower arrived.\* About 40 years ago, I became interested in my ancestral background, and then it lay dormant for some years. But now, in the last year or so, my interest has been rekindled. On both my mother's and father's side, my ancestors were farmers, as far back as we know, in Germany. My maternal grandmother, Barbara (Rieth) Lucker was born in Germany, and all of my great-grandparents arrived from Germany in the late 1800's.

My immediate family had a rough beginning, because my father was 15 years old when the height of the Great Depression happened in 1929, and finances were very scarce in the family. So he quit high school and got a job as a truck driver. He never completed high school. My mother was actually two years older than my father, and she completed high school. My mother and father had rather different personalities. My mother was very outgoing, you might say, a member of the popular crowd in high school, whereas my dad was rather reserved and a very low key guy. But anyway, they had a very happy marriage. Because of economic difficulties, it took some time for my dad to actually save up enough money to buy a farm. We lived with my grandmother for one or two years, and then we rented a farmhouse for a few years. During my first grade, my parents had saved enough money to buy a 30-acre farm for ourselves. Consequently, I changed schools in the spring of my first grade.

Getting back to my early childhood

\* The ship that carried a group of English families from England to Plymouth, Massachusetts, where they established the first permanent New England colony in 1620.

before my first grade, I had a rather tumultuous childhood. I became very sick at the age of two, and I was taken to the hospital in a very weakened condition. It was thought that I might lose my eyesight, and my eyes became crossed. My mother told me that I stayed in the hospital for two weeks. Fortunately, with surgery, I did get better, and my eyes became uncrossed. And this is what Helen actually mentioned in her talk last night. During this illness, to test the sight in my eyes, physicians held playing cards at various distances for me to identify, because I knew the numbers and the playing cards from watching my dad and my uncles play poker or pinochle every Sunday evening. Doctors concluded that I had essentially no sight in my left eye because of my illness. Jumping ahead many years later, ophthalmologists said that, in fact, I did not lose my eyesight because of the illness, but I was born with little sight in my left eye. I'm legally blind in my left eye.



85th birthday conference poster.

I had a couple other rather traumatic experiences. As I mentioned, we lived on a rented farm for a couple years. We had cows, and I loved milk, but the milk from the cows was not pasteurized – it was just straight from

the cows. Anyway, I got extremely sick once, at the age of maybe three or four. Because of my illness, an inspector came from the health department in the state of Michigan—and saw that the cows were diseased. My dad, as I recall, was told that he had to shoot the cows; he buried them on the farm. In these days, my father would not have been allowed to do that.

“ On both my mother's and father's side, my ancestors were farmers

I had another traumatic experience. We lived only about one or two miles from Lake Michigan. We often went there to swim. There were large sand dunes along the shores, in particular about five miles to the south of us. One evening, at about the age of four or five, I was wading with my cousins in Lake Michigan. My cousins were actually a bit younger than I was. I don't know what happened, but all of a sudden I was underwater, and I remember reaching up with my left arm. My Uncle Alan saw my arm, and rushed out and saved me. Fortunately, he knew artificial respiration, and so I was saved from drowning.

How long was it that you were underwater?

BB: It was a very close call. I was unconscious when he got me. If he hadn't known artificial respiration, I think I would have died.

Oh my God.

BB: So, yeah, I was very lucky. I can still vividly remember reaching up with my left arm as I was swallowing water and losing my consciousness.

Before we moved to the farm that my dad bought in the spring, when I was in the first grade, I was

hit by a car. School was out and I was crossing the highway. It was actually a very busy highway—the main highway going from St. Joe and Benton Harbor to Michigan City and Chicago. I remember it like it happened yesterday. The school was on the corner, and my friends were already across the highway heading home a little bit to my right and then farther down the road... I remember looking very quickly to the left and anyway, I got hit by a car. I don't know what happened after that – I was unconscious. My mother said I was thrown 25 feet.

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It was a very close call. I was unconscious when he got me.

Oh my gosh.

BB: And so my left shoulder, my left knee and my head were damaged. I had a concussion and a broken collarbone and shoulder. I woke up on a metal table in the hospital and I can remember waking up with all sorts of people around, looking at me on this cold grey metal table.

How old were you?

BB: I was six, or had barely turned six when I was hit by a car. So yeah, it was a rather traumatic childhood that I had.

Your birthplace is St. Joseph, I believe...

BB: Yes, I was born in St. Joseph, Michigan, which is about 25 to 30 miles north of the Indiana border along Lake Michigan. I think St. Joe had a population of about 6000 at that time.

Benton Harbor lies to the north. St. Joe and Benton Harbor were called twin cities, but Benton Harbor was a bit larger. Life was rather segregated at that time. St. Joe was almost all white, and Benton Harbor was largely black. Stevensville, where I grew up, and where my dad had the farm, is five or six miles south of St. Joe.

We had a 30-acre fruit farm. Our main crops were black raspberries, which are still my favourite fruit, sour cherries, grapes, and peaches. But we also grew plums, pears, sweet cherries, and vegetables, but not in as great a quantity as the fruits. At that time, fruits were sold in a way that is completely different from current practice. We would pick the fruit and put them in containers, for example, quart boxes for raspberries. (There were 16 quarts per crate.) Very early in the following morning, my dad would take the crates on his truck to the market in Benton Harbor. Buyers, often grocery store representatives, from all over, many from Chicago, would bid on the fruit. If there were a lot of farmers selling, you wouldn't get as much money for your fruit as when there were fewer trucks at the market. Of course, the selling price also depended upon the quality of the fruit. This was our primary source of income.

However, for grapes, Michigan Fruit Cannery and Mogen David Wine were the primary purchasers. They bought grapes in bulk, so we didn't need to put them in small packages.

Peaches were put in half-bushel or bushel baskets. My dad would again take them to the market. Almost all of the fruit was picked by my mom, my dad, and myself. Occasionally, we hired migrant or local workers to help, especially in the busy season, but for the most part, the fruits were picked by my parents and myself.

That brings me to another question. Can you tell us about the parental influence on you when you were growing up?

BB: There wasn't much direct influence; most of the influence was religious. My parents were somewhat strict. Everyone on my father's side of the family were Lutherans — both in Germany, and here in the US. At that time in the US, there were three primary branches, often called synods, of Lutheranism, chiefly depending on the origins of family members. Our church belonged to the *Lutheran Church, Missouri Synod*. (The name stems from the fact that the main headquarters were in Missouri.) Almost all of these churches

were German in background. Our local church had over 3000 members, and each Sunday there were two services in English and one in German. We went to church every Sunday morning. (I don't recall that we ever missed church on Sunday morning.) Through high school, my church had a strong influence on me.

The *The American Lutheran Church* was a denomination in which the church members were mostly of Norwegian descent. Those in *The Lutheran church in America* were primarily Swedish in background.

The Lutheran Church Missouri Synod is a very conservative church, much more so than the other two branches that I previously mentioned. I left the Lutheran Church Missouri Synod at about the age of 23, and my parents weren't happy about it.

Sunday dinner at the church?

BB: No, at home. Our Sunday dinner at noon was the main meal of the week. Generally, we tried to keep Sunday free from picking fruit, but it didn't always work that way.

What were the things that fascinated you as a child?

BB: When I was young, I often played cards with my mother, and, more generally, with my parents and friends. I liked model aeroplanes, and I made them myself out of cardboard and paper. One of my most embarrassing instances occurred when I was in kindergarten. At the school Christmas program, the kids sang Christmas songs on a stage. I had just learned how to make and fly model aeroplanes. So in the middle of one of the songs, I went to the front of the stage and proudly threw an aeroplane into the audience, much to the embarrassment of my mother. An electric train set, and collecting baseball cards and comic books were my primary hobbies later in my childhood.

Can you also tell us about your siblings?

BB: My sister Barbara is three years





Bruce Berndt

With siblings and family. (L to R) Berndt, Helen, brother-in-law Fred Nott (Helen's brother), Jane Nott (Fred's wife), daughter Kristin, brother-in-law John Mueller, sister Barbara Mueller.

younger than I am. She and her husband taught in Lutheran schools for many years. Barb's husband died five years ago, and she recently moved into a retirement home. I have always been close to my sister and her husband. We differed—we're on the opposite sides politically and religiously—but I've always refrained from bringing such topics up in conversations. So we've always had a good relationship despite these differences.

**Could you tell us about the primary schools that you attended?**

BB: Both of the schools that I attended for the first eight years of my primary education were two-room schools. Kindergarten through the fourth grade were in one room, and fifth through the eighth grade were in another room.

**So one single teacher would be teaching?**

BB: Yes, a teacher taught five grades in one room, and another teacher taught four grades in the "big room", as we called it.

**Were there any favourite teachers during that time?**

BB: At my first school, Stewart School, which I attended for a little less than two years, the teacher was very good at teaching handwriting. Consequently,

my handwriting is probably better than that of most people. Up through fourth grade, first at Stewart School and secondly at Evans School, my teachers were quite good.

After that, from fifth through seventh grade, I had an extremely poor teacher. During the warm months, we played softball during both the morning and afternoon recesses, and during the noon hour as well. Our teacher played with us, and the normal recess times were often stretched by as much as a half-hour. When I was in seventh grade, our teacher did inappropriate things. He was immediately fired, and his licence was taken away. In summary, I did not get a good education during those three years.

In the Lutheran church, at the ages of 13 and 14, children normally receive intensive religious instruction, and at the conclusion are "confirmed". One is required to learn a considerable amount of Biblical content and Lutheran teaching. My mother and the mothers of two of my second cousins arranged to carpool and drive us to St. Joe to attend the Lutheran school, where eighth graders received religious instruction for an hour or more each day. At the conclusion of the eighth grade, I was confirmed and became a member of the church.

As I previously related, our Lutheran church had over 3000 members. Consequently, the Lutheran school was large with over 450 children.

In my eighth grade, we had 48 students, whereas if I had stayed at Evans School, our eighth grade class would have had either six or seven members. My teacher at the Lutheran School was very good, and for the first time in my life I had homework.

**Was it because of the fact that you did not get good teachers for fifth through seventh grade that you decided to change the school?**

BB: Primarily, my parents wanted me to be confirmed in the Lutheran church. However, yes, the opportunity to receive a better education was also a large factor.

**What were your favourite avocations at that time, like music, sports or arts?**

BB: I liked baseball, but mostly we played softball. Baseballs are harder, nine inches in circumference, whereas softballs are twelve inches in circumference. I wanted to become a major league baseball player. It was only many years later that it became apparent that, unfortunately, I never could have become a baseball player, because I had hardly any sight in my left eye and consequently had limited depth perception. Whenever I faced a fast pitcher in either softball or baseball, I usually struck out. No one ever told me, "Well, you can't see as well as others and, you

know, that's why you strike out." Someone should have told me that I had talent in running. But this was something that I never thought about.

I also liked to play basketball, but for similar reasons, I was always uncomfortable on the basketball court.

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the idea of going to a large university just terrified me

I mentioned earlier that I was faster than most kids. I ran track all four years during high school, and I was much more successful at track than I was in other sports. Schools were classified as A, B, C, and D according to enrollment. St. Joe High School was in class B. During my senior year, the state track meet was held at Michigan State University. At that time in history all of the tracks were cinder tracks, and track events were measured in yards. I qualified in the quarter mile (440 yards) for the State meet. It rained very hard all day of the State meet, and most of the track was covered with water. Toward the end of the quarter-mile, I was in the lead, but I could see this guy from River Rouge slowly coming up. I was exhausted, and I threw myself across the finish line and finished first.

My teammate finished first in the 100 and 220-yard dashes. Another teammate placed in the long jump, the high jump, and the hurdles. We consequently won the state championship, which was a complete surprise. We had had no thoughts of ever winning a state championship; it was the farthest thing from our minds.

On the following weekend, an evening meet was held at the University of Michigan, where runners participated by invitation. In particular, I competed against the class A quarter-mile champion, and, in finishing first, I beat him. So I can say, indeed, that I was the Michigan High School champion in the quarter-mile.

**How were you as a kid? Did you like**

**to make pranks or very simple and sober?**

BB: I was very introverted and reserved. I don't recall much from my childhood about pranks.

**What were the early indications towards mathematics? Maybe people or books you admired at that time, which served as an inspiration for you?**

BB: I didn't have any. Only in the eighth grade did I really start to get an education. Generally, I liked all subjects. I remember liking geography, in particular. I finished third in the class of 48 at the Lutheran school, which I suppose in retrospect was not too bad because all the other kids in my class had a better education than I did up until eighth grade. I liked mathematics but I didn't really think of it as being a special subject when I got to high school.

In my freshman year I took woodshop—big mistake. I didn't have any skills in that area. I got three B's in my freshman year: two in woodshop and one in civics. After that, I got A's in all of my subjects. There were 154 students in our graduating class; I finished third again.

There weren't any subjects in which I particularly excelled. In my senior year I took physics, which I really enjoyed, and so decided to major in physics in college. I was very introverted, and the idea of going to a large university just terrified me. So, I went to the senior counsellor and told him that I'd like to attend a small college. He said that Albion College and Kalamazoo College were the two best small colleges in Michigan. I had run track on the Kalamazoo College campus, and I thought it was a rather unattractive college. I visited Albion College and decided to go there and major in physics.

**Any specific events or circumstances which led you to go into pure mathematics?**

BB: As I related, I entered Albion College, intending to major in physics. At

the end of the first year, it was clear that there were others in physics stronger than I was. In fact, my roommate—who was actually a year ahead of me—and one of my best friends were very good at physics. On the other hand, I was better at maths and enjoyed it more than physics during my first year. Both friends went on to get PhDs, one at Penn State and the other at Wisconsin.

It was clear that I liked maths better, so I switched to maths as my major. Most of my courses were from one professor, Keith Moore, who was a wonderful teacher and had a very strong influence on me. He played the oboe.\* I said to myself, "this is the kind of person I would like to be", to go back and teach in a small college like Albion College. I continued to take all the physics courses, but I was really better in maths than physics.

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I liked maths better, so I switched to maths as my major

**Among the mathematics courses, what was your favourite back then?**

BB: Well, let me tell you, we didn't have that many.

I liked real analysis and ordinary differential equations. We didn't have number theory or complex analysis, for example. When I took the SAT to go to graduate school, I was at a disadvantage because there were questions that I couldn't answer because I didn't have courses in those subjects.

**Were you in contact with your roommates after you graduated from Albion College?**

BB: As mentioned earlier, my freshman roommate, Roger Hooverman, majored in physics. He went to graduate school at Wisconsin, where I entered as a graduate student one year later. After Helen and I married, we

\* A long, slender wooden tube, widening out into a bell shape at the end. Attached to the wood are metal keys which are operated by the musician's fingers to open and close holes along the length of the tube. A standard oboe is 2 feet long.

often socialized with him and his wife. But after our PhDs, somehow, we lost contact with them.

The second physics major whom I mentioned earlier was Wilbur Hurst, who also ran track with me. He is still a close friend of mine. He lives in Damascus, Maryland, which is just a couple miles from Rockville, where our younger daughter Sonja lives. Often when Helen and I visit her and our two grandchildren, we visit with Wilbur and his wife. He worked for the National Institute of Science and Technology during his entire career.

I applied for graduate school at Michigan, Illinois, Wisconsin and Purdue. I was rejected at Illinois and Michigan, and I chose Wisconsin.

During your Wisconsin days, there might have been many professors that you took courses with, before deciding on your supervisor. How did you decide who you wanted to work with?

BB: At first, I didn't know the area that I should specialise in. I initially thought of differential equations. There were two or three main professors in the

area, but one of them had a cohort of four graduate students under him, and I was clearly on the outside.

I was a bit discouraged, but then I took a course in asymptotic theory of ordinary differential equations, which I really liked, and actually would do extra problems.

But the professor got upset with me, because he thought that I was trying to put the other students down by doing extra work. That discouraged me; I just did more problems because I liked to do the problems.

So I left differential equations and went to complex analysis. I had a good professor, R. Creighton Buck, who was a well-known researcher in complex analysis and wrote a popular book on complex analysis. He was willing to take me on as a graduate student.

In the spring semester of my third year, I took a course in modular forms from Rod Smart, because I was told the subject used a lot of complex analysis. I really liked the course, and so I asked Rod to be my advisor. In the following semester—this would have been the fall semester of my fourth year—I took a course in modular forms with applications to number theory, taught

by Marvin Knopp. This is where I learned about Ramanujan, the circle method, and partitions for the first time.

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I just did more problems because I liked to do the problems

So your move from Albion College to Wisconsin, especially because you mentioned not wanting to go to a big college—was it daunting?

BB: Yes, it was. In fact, during my first year there, I took analysis from Walter Rudin, who wrote a very popular text on analysis. I'll never forget his opening words in the class: "The only requirement for this class is plenty between the ears." He was a very demanding teacher, but I liked the course and I did okay in it.

In my first year, I also took a course in topology, which was very difficult for me. It was more abstract than anything



Bruce Berndt

A fuller family portrait. (L to R) daughter-in-law Eunita, son Brooks, granddaughters Kylie and Danalyn, daughter Kristin, Berndt, granddaughter Elise, Helen, grandson Benji, daughter Sonja, Anne (Kristin's daughter).



I'd ever taken. I got a B in the course just because of the generosity of the professor. But, I didn't like it at all. I've never cared for abstract mathematics—I've always liked concrete mathematics.

**How was your experience with Rod Smart as your mentor?**

BB: Smart was a methodical teacher, and gave very good lectures. There weren't very many of us in the course—maybe six or so. He has always been a very nice guy. I try to visit him every two or three years. He still lives in Madison with his wife Pat, and we keep in touch through email and Christmas letters.



Bruce Berndt

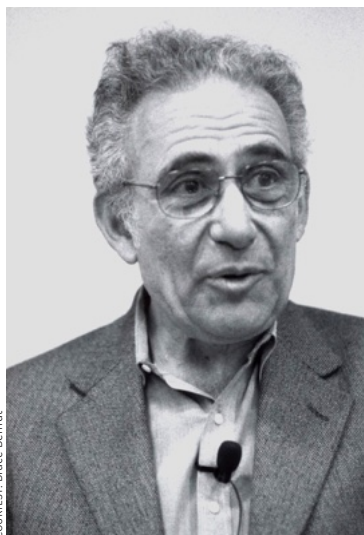
Rod Smart

**During your graduate school, do you remember any particular conferences or meetings that you went to?**

BB: Well, the only one I remember was in my last year. Smart was at the University of Glasgow on sabbatical that year. So, Marvin Knopp was my stepfather, you might say, but a nice stepfather. He received his PhD under Paul Bateman, a very well-known analytic number theorist, and the head of the maths department at Illinois. Knopp said, "Bruce, you must apply to Illinois; Paul Bateman has a great program in number theory." I wasn't interested at all in taking a job at a large university, but I said, "Okay, I'll apply to Illinois".

The AMS annual meeting that year was in Chicago. Knopp told me that I should go to the meeting and meet Paul

Bateman. I went and met Bateman on the first or second day of the meeting. He was surrounded by lots of people looking for jobs, and by professors—including Marvin Knopp. It was just before lunch, and so we all went to lunch together—about a dozen of us—to an extremely expensive restaurant. I didn't have much money, so I ordered a hamburger. A lot of the people, the professors especially, ordered drinks and other expensive items. At the end of the meal, Marvin Knopp got up and said, "It is going to be too complicated to divide this bill properly into who had what. So let's just split it even." I was furious. Here I was, a graduate student, paying for these expensive drinks of rich professors.



COURTESY: Bruce Berndt

Marvin Knopp (1933–2011)

Anyway, I did meet Paul Bateman, but that was all. After receiving my PhD in June, 1966, I went to the University of Glasgow for a postdoctoral appointment. In February, I received a letter from Paul Bateman, offering me a job as an assistant professor. It was a huge surprise, because I had completely forgotten that Marvin had urged me to apply to Illinois. During my final year at Wisconsin, I had been interviewed at some small colleges and received two or three offers. However, there was something about each of the colleges that I didn't like, and consequently I wasn't really anxious to accept any of these offers. In fact, at

that time, I didn't really know what I was going to do until Bateman's letter arrived. I thought, "I'll go to Illinois for just a couple years, find out what mathematics is about, get my feet on the ground, and be better prepared to go teach in a small college." Well, as you know, the rest of the story is: I never went to a small college.

The faculty at Illinois, especially in number theory, was very welcoming, and I enjoyed my research and teaching. There were a lot of courses—undergraduate courses that I had never taken. In particular, I never had taken courses in elementary number theory and complex analysis. Since there were many undergraduate courses that I had never taken, I signed up to teach them. In my first six years at Illinois up to my first sabbatical leave in 1973, I actually taught 23 different courses. The teaching load was two courses per semester. You have to take this with a little bit of a grain of salt, because some of the courses were not really different, but only variations of others—such as honours calculus and regular calculus, or differential equations for engineers and differential equations for mathematicians.



Bruce Berndt

Holding the slate on which Ramanujan scribbled his thoughts, if only to frantically erase them with his elbow, and then quickly refill it all over again with some more calculations.

**When did you first hear the name of Ramanujan, do you recall?**

BB: It was in Marvin Knopp's class. He mentioned Ramanujan when we were using modular forms to prove congruences for the partition function. He also discussed the Hardy–Ramanujan circle method, though we didn't use the circle method to prove anything in the course.

But your advisor, Rod Smart, was a student of Rademacher.

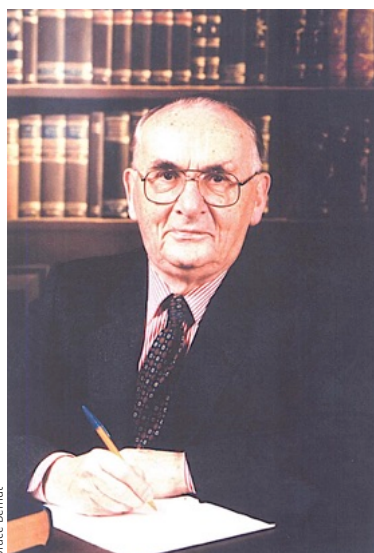
BB: No, a student of Joseph Lehner, who was a student of Rademacher.

So through him, do you recall hearing about Ramanujan?

BB: He might have mentioned Ramanujan, but I don't recall that he did.

And at that time, did you know anything about Ramanujan's life?

BB: No. I'm pretty sure that I didn't hear the word Ramanujan until Marvin Knopp's class.



Robert Rankin (1915–2001)

You also mentioned yesterday in the banquet about meeting Rankin in Glasgow. How was your lifelong association with him, with regards to Ramanujan in particular?

BB: I didn't have that much contact with Rankin in Glasgow, but I had a few conversations with him in his office. He was a very approachable and welcoming person, and he invited Helen and I to dinner four times. He was emphatic that I should go to tea every morning to meet people, *but not to talk about mathematics*. My close association with Rankin really started later in my life after I became fascinated with Ramanujan, mostly

mathematically, but also historically.

And then you wrote two books together.

BB: Yes. Over time, from both his beautiful papers on Ramanujan's life and notebooks, and also from personal correspondence, I learned a lot. He was an excellent collaborator.

Early on in your mathematical career, you mostly wrote single author papers until you started collaborating. I recall you mentioning afterwards about how wonderful collaborating is. So could you please say something about it?

BB: The first 35 papers that I published were single authored papers. When I was at the Institute for Advanced Study during my first sabbatical year, I got to know Chowla very well. He did not have an appointment at the Institute, but he lived in Princeton, and he loved the atmosphere at the Institute and talking with people there. He was a wonderful person, and we frequently talked about mathematics. I wrote two papers with him. It was wonderful writing papers with this very clever fellow; it was great.

I didn't think much about it at the time, about co-authoring papers, but after my experience with Chowla, I sought more collaborations. We occasionally invited Chowla for dinner, and he would always bring marzipan for our two children. Whenever he came, our two daughters would rush to the door to get marzipan.

One other aspect of your career as a professor of mathematics was guiding students, especially PhD students. I recall somewhere in 2012, you also received the graduate student mentoring award from the Center for Advanced Study in Illinois. How was your experience guiding PhD students?

BB: It was wonderful. In fact, it all started with Ron Evans. I remember I was standing in the hallway one day, I think it was before or after the number theory seminar, and Ron came up to me and asked me to be his thesis advisor. I was very surprised. I had taught the

beginning graduate course in analytic number theory, and Ron was a very creative, clever student—definitely the best in the course. Marvellous student! I had assumed that he would ask Paul Bateman to be his advisor. So when he asked me, I was totally shocked. This was perhaps in my third or fourth year at Illinois.



Paul Bateman (1919–2012)

Later when I started working on Ramanujan's notebooks, not surprisingly, I got stuck several times, and Ron bailed me out a few times with very clever solutions to problems that I couldn't solve, especially in "Chapter 14 of Ramanujan's second notebook".

From then on, I was fortunate to guide more graduate students. Of all the experiences I've had as a professional mathematician, the most rewarding to me has been advising graduate students. While I have been fortunate to have received some awards, and I'm very grateful for them, advising my graduate students is the highlight of my career, and the most satisfying.

And you have had, since then, more than 30 or 35 students.

BB: I've had 37. Hannah Burson was my 37th student.

And you still continue to supervise Likun Xie.

BB: I currently advise two students, Likun Xie and Raghav Bhat. I co-advise them with Alexandru Zaharescu. I also officially advise a



graduate student, Örs Rebák, at the Arctic University of Norway.

Before we move on to Ramanujan, can you talk a little bit about the various responsibilities or positions that you had when you were at Illinois, other than, of course, supervising students and teaching courses?

BB: Yes, I had a lot of committee assignments over the years. We alternated who would run the number theory seminar, and I was on various university committees. The one that I was most interested in was the library committee. I was on that committee for many years and sometimes chaired it. Of all the committees to which I had been assigned, the library committee was the one I enjoyed the most. Our library has a lot of old books and journals that most university libraries do not have. These are just gems, in my view. Preserving them and bringing in more maths books were very important to me, and these were my primary interests when I was on that committee. Our maths library is

regarded as one of the three best maths libraries in the country.

And how was your experience serving on all of these committees?

BB: Well, I don't remember much. I never had any serious difficulties with other faculty members.



COURTESY: Bruce Berndt

Emil Grosswald (1912–1989)

Coming to Ramanujan, I know this is something you may have been asked many times, how did Ramanujan enter in your mathematical career?

BB: It all began in my third year at Illinois when Paul Bateman asked me to referee two papers by Emil Grosswald. When you're the Head of the Department, like Bateman was, you have trouble keeping up with a large number of responsibilities. Also, heads take on more responsibilities than chairs, and often operate without advisory committees. Bateman did everything and made the decisions himself. So, he was especially busy, and thus asked me to referee Grosswald's papers.

In these papers, Emil Grosswald proved some formulas from Ramanujan's notebooks. He was at Temple University and evidently had a copy of the notebooks published by the Tata Institute. I accepted the papers for publication. One appeared in the *Nachrichten der Akademie der Wissenschaften, Göttingen*, and the other in *Acta Arithmetica*. At that time I didn't think much more about it.



Bruce Berndt

With students and postdocs. (L to R) Front row: Amita Malik (student), Berndt, Helen, Heekyoung Hahn (student), Soon-Yi Kang (student); second row: Byungchan Kim (student), Chadwick Gugg (student), Heng Huat Chan (student), Paul Bialek (student); third row: Song Heng Chan (student), Jaebum Sohn (student), Hannah Burson (student), Vicki Reuter (student), Atul Dixit (student), Doug Bowman (postdoc), Ae Ja Yee (postdoc); fourth row: Youn-Seo Choi (student), Tim Huber (student), Zhu Cao (student), Dennis Eichhorn (student), Armin Straub (postdoc).

During my sabbatical year at the Institute for Advanced Study in Princeton, one day in February, while I was working on transformation formulas for Eisenstein series, I discovered that I could use my methods and theorems to prove the formulas that Grosswald had proved. I was elated! It was a wonderful feeling—I could prove formulas of Ramanujan!



B.M. Wilson (1896–1935)

The Institute did not have the Tata Institute’s photocopy edition of the notebooks, but the Princeton University Library did. I checked them out, and I found a couple other results that I could prove, but a few thousand others that I couldn’t prove. It was a marvellous experience going through these pages of wonderful formulas. When I got home, the first thing I did was to order a copy of the notebooks from the Tata Institute in Bombay. They cost a total of \$25, including shipping in a wooden case. Can you beat that? \$25 for 3000 theorems?

At the conclusion of the following spring semester in 1975, I said to myself, just as a challenge, “Let’s try to prove everything in Chapter 14 of the second notebook”. This is the chapter in which the formulas that Grosswald proved are found, so I thought I had a head start. I worked on this for over a year. There were three formulas that I couldn’t prove, and Ron Evans was able to prove them. Thus, with Ron’s help, I completed the proofs of the entries in Chapter 14.

I hadn’t planned to do anything more. However, George Andrews

told me that when he discovered Ramanujan’s lost notebook in the spring of 1976 at Trinity College, Cambridge, he also found the notes that G.N. Watson and B.M. Wilson had accumulated in their efforts to edit the notebooks in the late 1920s and early 1930s. I wrote to the librarian at Trinity College, and she sent me a copy of these notes. They began with Chapter 2 of the second notebook. I then decided that I would go back to Chapter 2, and with the help of these notes, begin to systematically edit the notebooks. (Chapter 1 was on magic squares, and I felt I could handle that at any time.) After more than 20 years, with the help of many of my graduate students, I completed the work in five volumes published by Springer.

$$\begin{aligned}
 & \text{21. i. If } \alpha \neq \pi^2 \text{ and } n \text{ any integer,} \\
 & \{ \alpha \}^{2n-1} \left\{ \frac{1}{2} S_{2n-1} + \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) + 2c \right\} \\
 & - \{ \alpha \}^{2n-1} \left\{ \frac{1}{2} S_{2n-1} + \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) + 2c \right\} \\
 & = \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} + \pi^{2n-1} \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} \\
 & - \pi^{2n-1} \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} + 2c \text{ the last term} \\
 & \text{being } -\pi^{2n-1} \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} \text{ according as } n \text{ is even or odd.} \\
 & \text{ii. If } \alpha \neq \pi^2 \text{ and } n \text{ any integer, then} \\
 & \alpha^{2n-1} \left\{ \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) - \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) + 2c \right\} \frac{E_{2n-1}}{12n-4} \\
 & + \alpha^{2n-1} \left\{ \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) - \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) + 2c \right\} \frac{E_{2n-1}}{12n-4} \\
 & = \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} - \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} \\
 & + \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} - 2c \text{ the last term being} \\
 & (-1)^{2n-1} \frac{E_{2n-1}}{12n-4} \{ (-\alpha)^{2n-1} + \alpha^{2n-1} \} \text{ according as } n \text{ is} \\
 & \text{odd or even.} \\
 & \text{iii. If } \alpha \neq \pi^2 \text{ and } n \text{ any integer, } \frac{1}{2} \{ (-1)^{2n-1} + (-1)^{2n} \} \\
 & + \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) - \frac{1}{12n-4} (e^{2\alpha} + e^{-2\alpha}) - 2c \} =
 \end{aligned}$$

A page from the second Notebook.

When my coauthors, who were mostly my graduate students, and I completed the proofs of everything in a particular chapter, we would publish a paper on the chapter in a journal. One day, when I was in my office, Professor Heini Halberstam phoned me and asked me to come to his office. I went across the street to his office, and he introduced me to Walter Kaufmann Bühler, the main mathematics book editor at Springer-Verlag. Kaufmann Bühler had come to visit Halberstam about the publication of selected papers of Hua, Loo Keng, which were being edited by Halberstam and eventually were to be published by Springer-Verlag in 1983. Kaufmann Bühler said to me, “Professor Halberstam

has told me about your work on Ramanujan’s notebooks. Would you consider publishing your work on Ramanujan’s notebooks in books for Springer-Verlag?” I had not previously thought of publishing my work in a book[s]. I said “Okay,” and I soon signed a contract.

The first book was in 1985.

BB: Working with Kaufmann-Bühler was very pleasant. We were really on the same page, so to speak.

Sometime during the Princeton days, when you came to know that your general theorem can actually prove a lot of theorems from Ramanujan’s work, perhaps by then you may have also read the biography of Ramanujan and became aware of his life?

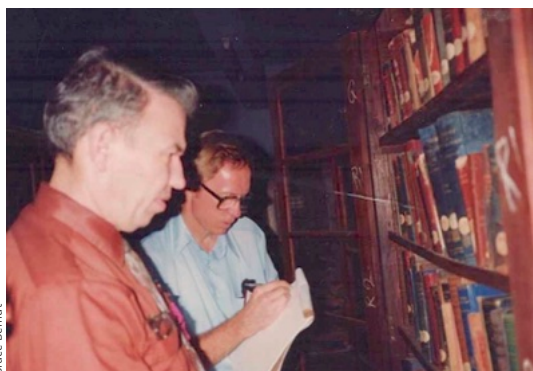
BB: Yes, as I worked through Ramanujan’s mathematics, I became more and more interested in his life and influence. I can’t give a starting date, but it was early in my work. I started reading Hardy’s book, *Ramanujan: Twelve Lectures Suggested by His Life and Work*. However, my visit to India in 1983–1984 was the primary influence.



G.N. Watson (1886–1965)

And you completed the five volumes on the notebooks somewhere in 1998?





Bruce Berndt

At work with George Andrews



Bruce Berndt

In his office with Alexandru Zaharescu.

BB: Yes, that's correct.

Then you decided to collaborate with George Andrews. How did that collaboration come up?

BB: I had already learned about much of the content of the lost notebook from George Andrews' papers. In 1988, *The Lost Notebook and Other Unpublished Papers of Ramanujan* was published in connection with the centenary of Ramanujan's birth. I was given a copy. In fact, it was an honour for me to receive one of the first copies that was published, as I was in India for the centenary. When I saw all these wonderful formulas, I was anxious to start proving them, although, of course, many had been proved by George Andrews and his colleagues and students.

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as I worked through Ramanujan's mathematics, I became more and more interested in his life

But there were a number of things there that I knew that Andrews probably was not strongly interested in, but I was. I asked George, in 1999 or earlier, if he would be interested in writing books on Ramanujan's lost notebook, analogous to those that I wrote on the earlier notebooks, and he was quite ea-

ger to do so. He thought it might take two volumes, but in the end it took five volumes, with the last being in 2018.

We sort of proceeded in the same way: if a topic was completed, then, we wrote it up, and revised or updated it as necessary. We didn't proceed chronologically, we just did so by topic. It was good working with George. We collaborated very nicely. We agreed on everything, and we had no disagreements. It took us 20 years to complete the task.

That's great.

BB: Yes, everything went very well.

How far do you think you have been able to accomplish the mission of a comprehensive and complete study of Ramanujan's work?

BB: Although essentially everything has been proved in the earlier notebooks and lost notebook, we really don't understand how Ramanujan derived a lot of his results, especially, for me, his modular equations of larger degrees, seven or more. We have no idea how Ramanujan proceeded. I have spent hours and hours trying to prove these modular equations of Ramanujan without using the theory of modular forms.

I knew fairly early that I could use modular forms to prove these modular equations, but I didn't want to do that because Ramanujan didn't proceed in this way at all. So here, actually, I used Watson's work a lot. I don't recall exactly where Watson ended and I began, but I think for degree seven already, it was beginning to

get difficult. I have never been able to figure out how Ramanujan would have argued. There are ideas of Ramanujan which we are missing, and which I would love to have, to prove these modular equations.



Bruce Berndt

Front elevation of Ramanujan's home in Kumbakonam when Bruce Berndt first visited there.

Just recently, in the last few months, I looked at degree eleven again. There is something he knew that I don't know. He has differential equations, first order equations, involving the *multiplier*, and the formulas suggest that Ramanujan used them, but it doesn't really fit in with the forms of the modular equations. I just can't figure this out at all. It's a huge mystery. So for all these higher degrees, I had to use modular forms. And for none of them do I have a clue as to how Ramanujan argued. It's a huge mystery.

I remember when I had been to your home for the first time, I think sometime in the fall of 2006, I had asked you what you would do if you got a chance to meet Ramanujan and you had said you would ask him what ideas he used to prove his beautiful theorems.





With Richard Askey and George Andrews.



Well at the back of Ramanujan's home.

BB: Yes. I would ask him how to prove his modular equations. However, he had lots of other results, for example, continued fractions and asymptotics, that would be the focus of my next questions.

Before the next question: was your first trip to India, in the year 1987, for the centenary?

BB: No. It was in 1983-1984.

And were you able to meet Mrs. Ramanujan?

BB: Yes, I actually met her twice altogether. It was a great thrill for me to meet her. She would get very excited talking about her late husband, although she didn't speak English. She had raised two boys who were sons of a

couple in her neighbourhood who died, and she took them in as her own. One of them was not close to her anymore, but the other one was. He worked in a bank in Madras, and he served as the translator for me. He really took care of his adopted mother in the last several years of her life. A very nice fellow. Unfortunately, he died not too many years after he served as my translator. I would have liked to write to him more and ask more questions.

So we all know Hardy's favourite formula from Ramanujan, or what he called as the most beautiful identity. If I were to ask you, what would be your favourite formula in Ramanujan's work, if at all, you can narrow it down to a single one?

BB: I actually have not really thought

much about this. Each one of Ramanujan's identities is beautiful, and the two formulas that I talked about in my lecture this morning are amazing, and I would love to know why Ramanujan derived these formulas. I like his results very much in his cubic theory of theta functions. I like his integrals of Eisenstein series; I don't know how he ever thought of these. The Rogers-Ramanujan identities are certainly among my favourites.

It's hard to pick a single one.

BB: His work, his elementary work in the theory of equations is really neat. It's elementary, but it's very creative. So you can see that, early on, even before he went to England, he had lots of new and really beautiful results, and just in elementary areas.



At Atul's wedding in India.



COURTESY: IIT Gandhinagar

Audience at the public lecture at IIT Gandhinagar.

Right. Sitting at this point at the conference organized in honour of your 85th birthday, along with George Andrews' 85th birthday, how do you retrospect on the course of your life that was decisively driven by the spirit of Ramanujan?

BB: Beginning in February of 1974, my life was changed forever. And I'm extremely lucky. Otherwise, I never would have reached this point in terms of having a celebration of my birthday like this. I owe all of this to Ramanujan. It's very humbling. I wouldn't have ever received the fame and respect that I have now. I often feel that it's really undeserved, because if it weren't for Ramanujan, I wouldn't be talking with you now.

That's your modesty.

BB: Really, I'm just lucky. It's fortunate that the kind of mathematics that Ramanujan did is the kind that I really like. I'm very classically oriented, right

from the beginning in my thesis, in which I read these wonderful papers of Chandrasekhar and Narasimhan. I think Rod just mentioned one day that you might find these papers interesting to read. And then I saw that I could prove generalizations and similar kinds of results. I think this is part of how I got the job in Illinois—because I came up with the thesis problems on my own. My guess is that in their letters for me, Marvin Knopp and Rod Smart emphasized that I found my own thesis problems and did my thesis work without the help of anyone else.

My thesis is in very classical analytic number theory, and this is the kind of stuff I really like. So, I was especially attracted to Ramanujan's work. I was extremely lucky that my work took me to Ramanujan's work. George Andrews is a giant in mathematics. It's very humbling to celebrate a birthday with him.

Of course, it's very well deserved, and we really have benefited so much.

BB: Thank you very much. I've gotten to know not only my wonderful students, but I've gotten to know many nice people. We really work in an area of camaraderie and respect.

I mean, just as you said Ramanujan changed the course of your life, you changed our lives when we started our graduate studies with you.

BB: I hope it's really good.

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Each one of Ramanujan's identities is beautiful

Yeah, it's been really wonderful since the fall of 2006, when I asked you to supervise my thesis and you took me as your student. That was really the best thing that happened to me.

BB: Students came from a variety of different backgrounds, and they have been quite different, so I have had to work with them in different ways. My relationships with all of them, as far as I know, have been good.

Some worked more or less on their own, while for others I needed to work more closely with them to finish their thesis. But in every case, I felt it was a good experience. In other words, I don't regret the paths we took together.

My last question is, I guess a part of it is related to the previous one too. How would you express

your sentiments as to what exactly about Ramanujan drew you dedicating your career towards showcasing his life and work?

BB: Well, you know, just like in music—say the music of Mozart or Beethoven or Sibelius or Brahms—there's a beauty to it. That's the best way to describe Ramanujan's mathematics—there's a beauty to it. There is also symmetry in much of Ramanujan's mathematics. Ramanujan loved symmetry. The symmetry of Ramanujan's work was emphasized in a couple of the lectures at the meeting.

His work is beautiful, and, like visual art or music, it is one of the pleasures and values of life. Ramanujan didn't have any ugly mathematics. It was all beautiful. When he tried to find an approximation, the path to it was beautiful, whether it be through an asymptotic formula or some sort of clever device or something you could derive from an equality. Creativeness, cleverness, beauty. These are, I guess, the best words to describe Ramanujan.

Wow, that's very well put. What a wonderful interview this has been. Thank you so much. ■